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THE GROUNDWATER LEVEL CHANGING PROCESSES MODELING IN 2D AND 3D FORMULATION

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The objective of this study was to develop a mathematical model to *determine the tendency of the groundwater level changes under the influence of external factors to prevent environmentally hazardous impacts and emergency situations. Mathematical methods (analytical solution of differential filtration equations involved the computer program Maple) - for creation the groundwater level changes model, methods of ecological and economic assessment and comparative analysis - for the identification of groundwater level impact important factors and groundwater level impact on the environment, balance method - for assessing the groundwater level changes. The mathematical model in 2D formulation works from any value of the initial* groundwater level. *The value of* groundwater level *changing at constant evapotranspiration has been obtained, which has been visualized by calculations for limited areas of the Kharkiv territory. Three-dimensional modelling of groundwater level changing in contrast to two-dimensional allows to take into account the dependence of evapotranspiration on the presence of artificial coverings on the soil surface, which are located unevenly and have different filtration coefficients, which causes corresponding groundwater level changes of urban areas. The nature of groundwater level changes under the influence of external factors has been determined. The necessity to create three-dimensional mathematical models to describe groundwater level changes and improve forecasts of their changes have been identified. A three-dimensional mathematical model of urban groundwater level changes, such as atmospheric water infiltration, additional groundwater replenishment, transpiration, evaporation, evapotranspiration, and groundwater abstraction has been developed. The boundary conditions of the three-dimensional mathematical model have been formulated.*

Keywords: additional groundwater replenishment, environmental safety, evapotranspiration, flooding, groundwater level, mathematical model.

INTRODUCTION

For the cities sustainable development, the construction protection against dangerous groundwater level (GWL) rising and flooding, it is necessary to correctly assess the existing hydrogeological conditions and to predict them with the proper accuracy (1). The main task is to determine the tendency of the groundwater level changes under the influence of external factors to prevent environmentally hazardous impacts and emergency situations. Technogenic activities lead to increased risks of emergencies of technogenic and natural origin. Therefore, the researches of assessing the readiness of civil defence units to act during emergencies of various nature have been conducted (2, 3, 4).

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The groundwater flow issues has been discussed by many scholars with various aspects, such as Klute (5) has reduced the diffusion equation to the ordinary differential equation and applied the method of direct integration and iteration of the obtained equation, Verma (6) has obtained the solution of an equation describing the one-dimensional supply of groundwater for constant diffusivity and linear conduction by the Laplace transform. Prasad et al. (7) have developed the numerical model to simulate the flow of moisture through unsaturated zones by the finite element method. Desai (8) has obtained the composite expansion solution for recharging groundwater in a vertical direction. Shah K. and Kunjan T. (9) have gained solutions of the Burger equation to describe the one-dimensional supply of groundwater by propagation in porous media. Joshi et al. (10) have obtained a solution of the equation for one-dimensional vertical groundwater supply by a group theoretical approach. Nasseri et al. (11) have investigated the solution of the advection-diffusion equation on the basis of the simplified Brooks-Corey model for soil conductivity and diffusivity.

The Koohestani N. has taken into account only natural groundwater sources and based on these data has made the groundwater balance and GWL changing forecast (12).

In the S. P. Pathak, T. Singh paper (13) it has got differential equation of one-dimensional groundwater supply according to Dupuis' assumption. Three cases with corresponding boundary conditions and different slopes of the impenetrable slope limit have been discussed.

The quantitative model of groundwater flow has been simulated by (14) for developing aquifer balance element analysis scenarios, explaining conditions of droughts, definition of prohibitive extraction policies and analysing the qualitative models.

The problem of mathematical modelling of issues associated with changes in the groundwater regime, dealt with such Ukrainian and Russian scientists as Yakovlev E.A. (15), Telyma S.V. (16), Kremez V.S. (17), Zolotarev N.V. (18), Vengersky P.S. (19) and others.

The different scholars have been considered the impact of flood hazard including topographic surveying techniques (20), hydrologic modelling (21, 22), geospatial techniques (23, 24).

The management models of groundwater have been treated by (25,26) and others.

In previous papers of the authors (27-31) it has been established and proved that in large cities the influence of technogenic factors of groundwater replenishment in several times higher than natural. Therefore, it is important to take into account natural and technogenic factors of groundwater influencing, to create mathematical models and forecasts to include it. Two-dimensional and three-dimensional model simulating the groundwater level changing processes will assume more clearly and objectively consideration of influencing GWL factors parameters change in long-term predicting. This will help to prevent emergencies caused by flooding.

MATERIALS AND METHODS

On the basis of the Muftakhov A. Zh. equation the mathematical model has been developed, which allowed to obtain the solution of the formulated problem in a closed analytical form (in the form of series). It has been visualized the results and confirm the previously obtained data by the author of the impact of additional replenishment on the groundwater level using a traditional engineering approach (29).

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In order to simulate the mathematical model, differential equations and the appropriate boundary conditions have been chosen to characterize the GWL changes (28).

The following assumptions were accepted:

- the GWL changes have the stable character, as evidenced by the data of long-term research of the Geological Party in 3 regime wells in Kharkiv (28);
- sites with homogeneous hydrogeological conditions along one of the directions were considered, which allows to use the equation of flat filtration.

THE GROUNDWATER LEVEL CHANGING PROCESSES MODELLING IN 3D FORMULATION

In many cities of Ukraine, the artificial coverings and structures occupied large area that prevent natural precipitation infiltration, evaporation and transpiration processes. Therefore, the groundwater level changes modelling it has been taken into consideration the existence of such areas partially covered with artificial surfaces, where the natural and technogenic factors influence will occur only on the undeveloped surface of this area (Fig. 1).

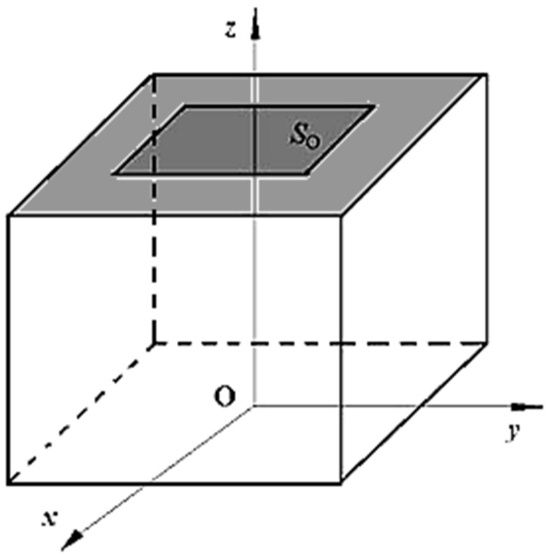

Figure 1. Calculation area for determining the GWL

Due to the fact that the conductivity in anisotropic soils in different directions is diverse, if the structure of the porous medium is such that it has a higher conductivity in one direction than in others, there is the need to take into account GWL changes in the three-dimensional modelling (1, 18). To predict GWL changes, the mathematical model has been developed to takes into account infiltration of atmospheric water, additional groundwater replenishment, transpiration, evaporation, evapotranspiration and groundwater abstraction.

The GWL changes have the stable character, as evidenced by the data of long-term research of the Geological Party in 3 regime wells in Kharkiv (28). In contrast to the research (31), this paper considers the issue of predicting the GWL changes in three-dimensional formulation.

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Consider the equation of filtration pressure in the form

$$\gamma^2 \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \,, \qquad [1]$$

where h is the groundwater level, x, y, z be coordinates have shown at Fig. 1, γ is the anisotropy coefficient.

The boundary conditions for equation (1) are formulated, which take into account the presence of artificial coverings, infiltration, evaporation and transpiration, as well as the effect of evapotranspiration. These conditions are corresponded to the values of the unknown function, or its normal derivative at the boundaries of the computational domain. The rectangular parallelepiped has been assumed as the computational domain. The lower and upper faces of this parallelepiped are rectangles S with sides [$2a$,$2b$]. The height of the parallelepiped has been denoted as L. Let S_0 be the square with sides [$2l$,$2l$], which is located in the centre of the upper face.

Suppose that on the domain $S_1=S\backslash S_0$ there are the influence of natural and technogenic factors on GWL changes; while in the area S_0 the impact on the groundwater level does not occur due to the presence of artificial surfaces. The following boundary condition that characterizes the presence of artificial coverings are formulated:

$$\left. \frac{\partial h}{\partial y} \right|_{S_0} = 0 \cdot \qquad [2]$$

On the site $S \setminus S_0$ there are infiltration, water abstraction, transpiration and evaporation, so it was obtained

$$\left. \frac{\partial h}{\partial y} \right|_{,S_1, y=0} = f + s - g - d - k \,,$$

where f – the additional groundwater replenishment (profitable part of groundwater balance); s – the groundwater replenishment by atmospheric waters (profitable part of groundwater balance); g – intensity of transpiration (expenditure part of groundwater balance); d – evaporation rate (expenditure part of groundwater balance); k – the groundwater extraction (expenditure part of groundwater balance).

Let turn to the conditions that take into account evapotranspiration. Since the GWL changes and their distribution are local, and modelling is carried out for limited areas of urban territory (industrial facilities, buildings, etc.), with homogeneous hydrogeological conditions, it could be assumed that the lateral inflow and outflow are equal to each other.

Thus, such the boundary value problem has been formulated to determine the unknown function i $h(x, y, z)$: it is necessary to find the solution of differential equation [1] under the following boundary conditions:

$$\left. \frac{\partial h}{\partial y} \right|_{S_0} = 0 \,,$$

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$$\left.\frac{\partial h}{\partial y}\right|_{S_1, y=-L} = f + s - g - d - k, \quad \left.h\right|_{y=0} = 0,$$

$$\begin{cases} \left.\frac{\partial h}{\partial x}\right|_{x=l+a} = e_1(y), & \left.\frac{\partial h}{\partial x}\right|_{x=-l-a} = e_1(y), \\ \left.\frac{\partial h}{\partial z}\right|_{z=l+b} = e_1(y), & \left.\frac{\partial h}{\partial z}\right|_{z=-l-b} = e_1(y). \end{cases}$$

In these equations according to (Vengersky, 2017), it was obtained

$$e_1(y) = \frac{2}{1+(y/y_{50})^{\tau}}, \quad [3]$$

where τ is the relative variability of potential transpiration; y_{50} is the parameter that characterizes the height of capillary water absorption; y is the depth where the absorbing moisture pressure occurs. In the calculations (29-32), the value of τ is τ = 2,2. In further calculations, it is assumed that y_{50} = 3, i.e. it was assumed that $L = 6$ m. If evapotranspiration was not taken into account, the value L discussed separately. The initial level has been taken as the starting point, $\left.h\right|_{y=0} = 0$.

Note, that it is impossible to construct the single system of basic functions for this boundary value issue with inhomogeneous boundary conditions at the sixth boundary. Therefore, the paper proposes to look for an unknown function $h(x, y, z)$ in the form of the sum of three terms

$$h(x,y,z) = h_1(x,y,z) + h_2(x,y,z) + h_3(x,y,z).$$

For each function $h_i(x, y)$, i = 1,2,3 he own boundary value issue is corresponded, and in each of these problems there are homogeneous boundary conditions, which makes it possible to build systems of independent basis functions. This way not only allows us to build the solution of the formulated boundary value issue, which takes into account the presence of artificial coverings, infiltration, evaporation and transpiration, as well as the effect of evapotranspiration, but also to investigate the effects of artificial coverings and evapotranspiration.

The boundary value issue for the function $h_1(x,y,z)$ describes the presence of artificial coverings, infiltration, evaporation and transpiration, but does not take into account the effect of evapotranspiration depending on the depth. This issue was formulated as follows:

$$\gamma^2 \frac{\partial^2 h_1}{\partial y^2} + \frac{\partial^2 h_1}{\partial x^2} + \frac{\partial^2 h_1}{\partial z^2} = 0 \quad [4]$$

$$\left.\frac{\partial h_1}{\partial y}\right|_{S_0} = 0,$$

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$$\left.\frac{\partial h_1}{\partial y}\right|_{S_1, y=-L} = f+s-g-d-k\,,\;\; h_1\Big|_{y=0} = 0$$

$$\begin{cases} \left.\dfrac{\partial h_1}{\partial x}\right|_{x=l+a} = 0, & \left.\dfrac{\partial h_1}{\partial x}\right|_{x=-l-a} = 0, \\ \left.\dfrac{\partial h_1}{\partial y}\right|_{z=l+b} = 0, & \left.\dfrac{\partial h_1}{\partial z}\right|_{z=-l-b} = 0. \end{cases}$$

For the function $h_2(x,y,z)$ was obtained such the boundary value issue

$$\gamma^2 \frac{\partial^2 h_2}{\partial y^2} + \frac{\partial^2 h_2}{\partial x^2} + \frac{\partial^2 h_2}{\partial z^2} = 0 \qquad [5]$$

$$\left.\frac{\partial h_2}{\partial y}\right|_{S_0 \cup S_1, y=-L} = 0\,,\;\; h_2\Big|_{y=0} = 0\,, \begin{cases} \left.\dfrac{\partial h_2}{\partial x}\right|_{x=l+a} = e_1(y), & \left.\dfrac{\partial h_2}{\partial z}\right|_{z=-l-a} = e_1(y), \\ \left.\dfrac{\partial h_2}{\partial y}\right|_{y=l+b} = 0, & \left.\dfrac{\partial h_2}{\partial z}\right|_{z=-l-b} = 0. \end{cases}$$

Similarly, for the function $h_3(x,y,z)$ it was obtained

$$\gamma^2 \frac{\partial^2 h_3}{\partial y^2} + \frac{\partial^2 h_3}{\partial x^2} + \frac{\partial^2 h_3}{\partial z^2} = 0 \qquad [6]$$

$$\left.\frac{\partial h_3}{\partial y}\right|_{S_0 \cup S_1, y=-L} = 0\,,\;\; h_3\Big|_{y=0} = 0\,,$$

$$\begin{cases} \left.\dfrac{\partial h_3}{\partial x}\right|_{x=l+a} = 0, & \left.\dfrac{\partial h_3}{\partial x}\right|_{x=-l-a} = 0, \\ \left.\dfrac{\partial h_3}{\partial z}\right|_{z=l+b} = e_1(y), & \left.\dfrac{\partial h_3}{\partial z}\right|_{z=-l-b} = e_1(y). \end{cases}$$

Using the method described in (Vengersky, 2017), the following solutions were obtained of boundary value issues (4)-(6)

$$h_1^{mn} = E^{mn} \cos\frac{\pi m x}{2(l+a)} \cdot \cos\frac{\pi n z}{2(l+b)} \cdot \sinh \lambda_{mn}(y+L), \quad \lambda_{mn} = \frac{1}{2\gamma}\sqrt{\left(\frac{\pi m}{l+a}\right)^2 + \left(\frac{\pi n}{l+b}\right)^2}\, m = 1,2,...$$

$$h_2^{mn} = F^{mn} \sin\frac{\pi\gamma(0.5+m)(y+L)}{L} \cdot \sin\frac{\pi(0.5+n)z}{(l+b)} \cdot \sinh \lambda_{mn} x, \quad m,n = 0,1,2,...$$

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$$h_3^{mn} = G^{mn} sin\frac{\pi\gamma(0.5+m)(y+L)}{L}\cdot sin\frac{\pi(0.5+n)x}{(l+a)}\cdot sinh\lambda_{mn}z, \quad m,n=0,1,2,...$$

$$h(x,y,z) = h_1(x,y,z) + h_2(x,y,z) + h_3(x,y,z)$$

$$h_i(x,y,z) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h_i^{mn}(x,y,z), \quad i=1,2,3\ ... \qquad [7]$$

THE GROUNDWATER LEVEL CHANGING PROCESSES MODELLING IN 2D FORMULATION

For GWL changing modelling, the equation of filtration pressure in the case of flat filtration has been considered

$$\frac{\partial^2 h}{\partial x^2} + \gamma^2 \frac{\partial^2 h}{\partial y^2} = 0$$

which can be the basis for creating a mathematical model for describing the changes of GWL, which can take into account the factors of artificial coverings and evapotranspiration.

In this the paper proposes to look for the unknown function $h(x,y)$ in the form of the sum of two terms

$$h(x,y) = h_1(x,y) + h_2(x,y).$$

There for each function $h_i(x,y)$, i=1,2 the own boundary value problem is corresponded, and in each of these issues there are homogeneous boundary conditions, which makes it possible to build systems of independent basis functions.

The solution of this 2D problem is obtained in similar way as for 3D problem beforehand.

RESULTS AND DISCUSSION

The mathematical models in 2D and 3D formulation work from any value of the initial GWL.

The first step was to define the requisite number of members in series (7) for evaluating the unknown functions with given accuracy. The convergence is established when numbers of members in series were equal to 4 both for m and n. Such high convergence is achieved due to the presence of exponential factors in the denominators similar to

$$\pi k \cosh(\pi k l(a+l)),\ k=1,2\ldots$$

In following, the next initial data were chosen: l=500m, a=b=1000m, L=6m, f, s, g, d, k - the additional groundwater replenishment (profitable part of groundwater balance); the groundwater replenishment by atmospheric waters (profitable part of groundwater balance); transpiration rate (the debit of groundwater balance); evaporation rate (the debit of ground-

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water balance); the groundwater extraction (expenditure part of groundwater balance) respectively.

The GWL changing value at constant evapotranspiration has been gained in 2D and 3D formulations and presented by calculations for limited Kharkiv territories on the Fig. 2, which shows the function $h_1(x,y)$. In this case it was supposed that value of GWL changing were calculated at y=3 and different values of z.

By number 1 the values obtained with usage of 2D model are denoted, numbers 2, 3 ,4 correspond to values $z=(l+a)\frac{\pi}{4}$, $z=(l+a)\frac{\pi}{6}$ and z=0.

From these results one can concluded that values of GWL changing obtained at 3D simulation are less than ones obtained as the result of calculations carried out according to the two-dimensional theory. Therefore, data of 2D simulation can be used as an upper bound for GWL changing estimation. GWL changing at other values of y have demonstrated the similar behaviour. These data are shown at Fig. 2.

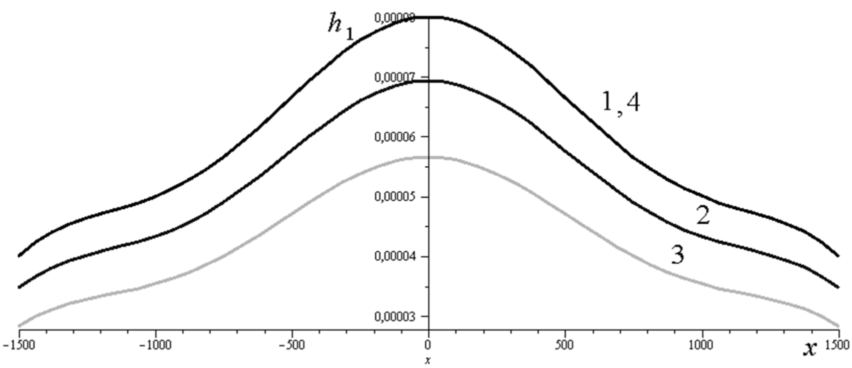

Figure 2. GWL changing at other values of y.

GWL changing at y= -2 and different values of z have presented at Fig. 3.

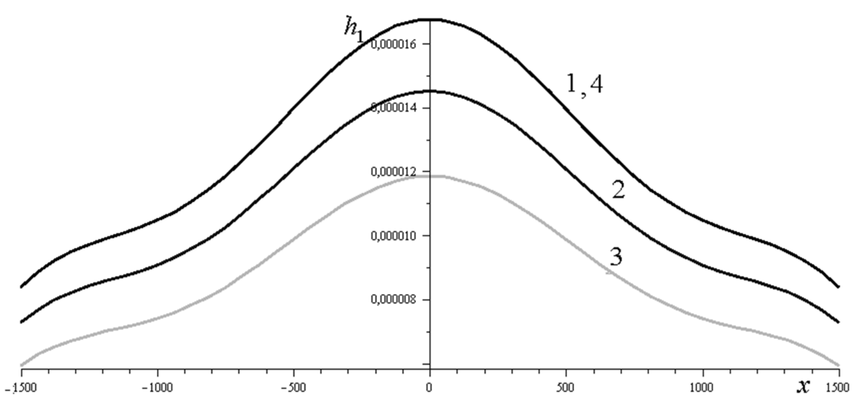

Figure 3. GWL changing at y= -2 and different values of z.

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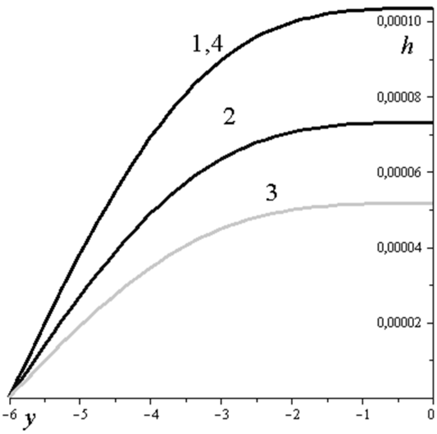
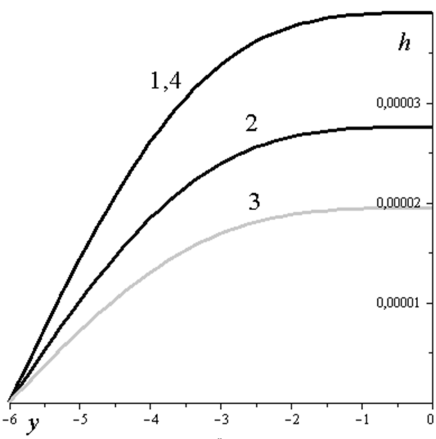
The values of GWL changing during variable evapotranspiration have been also obtained, which have shown in Fig. 4. Here numbers 1, 2, 3 correspond to values $z=(l+a)\frac{\pi}{2}$, $z=(l+a)\frac{\pi}{4}$, $z=(l+a)\frac{\pi}{6}$. By number 4 the results of 2D simulations are marked.

As before, these results have demonstrated that values of GWL changing obtained at 3D simulation are less than ones obtained as the result of calculations carried out according to the two-dimensional theory. Therefore, data of 2D simulation can also be used as an upper bound for GWL changing estimation. In figure 4a the data are obtained at $x=(l+a)\frac{\pi}{2}$ and different value of z, while in figure 4b the data are shown that were obtained at $x=(l+a)\frac{\pi}{4}$.



a



b

Figure 4. GWL changing with variable evapotranspiration.

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Three-dimensional modelling of GWL changing in contrast to two-dimensional allows to take into account the dependence of evapotranspiration on the presence of artificial coverings on the soil surface, which are located unevenly and have different filtration coefficients and caused appropriate GWL changes of urban areas.

CONCLUSION

The nature of GWL changes under the influence of external factors has been determined. It has been established and proved that in large cities the influence of technogenic factors of groundwater replenishment in several times higher than natural. Therefore, it is important to take into account natural and technogenic factors of groundwater influencing, to create mathematical models and forecasts to include it.

Two-dimensional and three-dimensional model simulating the groundwater level changing processes will assume more clearly and objectively consideration of influencing GWL factors parameters change in long-term predicting.

The necessity to create three-dimensional mathematical models to describe GWL changes and improve forecasts of their changes has been identified. A three-dimensional mathematical model of urban GWL changes, such as atmospheric water infiltration, additional groundwater replenishment, transpiration, evaporation, evapotranspiration, and groundwater abstraction has been developed. There have been formulated the boundary conditions of the three-dimensional mathematical model. The analytical solution of differential filtration equations involved the computer program Maple for creation the groundwater level changes model has been solved.

In the case of three-dimensional modelling of groundwater level changing processes, it has been obtained results with the lower rise in groundwater levels than the result of calculations carried out according to the two-dimensional theory due to the taking into account parameters changes in different sections. Therefore, data of 2D simulation can be used as an upper bound for GWL changing estimation and 3D simulation for more accurate evaluation.

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