March 5-6, 2024
Ankara, Türkiye

7. INTERNATIONAL ANKARA MULTIDISCIPLINARY STUDIES CONGRESS

PROCEEDINGS BOOK

EDITED BY
ASSOC. PROF. DR. HASAN ŞAHİN

ISBN - 978-625-8254-40-2
www.izdas.org

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Issued: 23.03.2024
ISBN: 978-625-8254-40-2

CONGRESS ID

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DATE-PLACE

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EDITORS

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EVALUATION PROCESS

All applications have undergone a double-blind peer review process

TOTAL NUMBER OF PAPERS: 355

THE NUMBER OF PAPERS FROM TÜRKİYE: 169

OTHER COUNTRIES: 186

PARTICIPANT COUNTRIES (29):

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CONTENT

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SLOSHING IN LIQUID-FILLED CYLINDRICAL-CONICAL SHELLS

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ÖZET

Akışkan salınımları konusu, havacılık, denizcilik, inşaat makine mühendisliği, nükleer mühendislik dahil olmak üzere çeşitli endüstriyel sektörler için zorluk teşkil etmektedir ve aynı zamanda fizikçiler ve matematikçiler için de karmaşık bir görevdir. Sıvı salınımları, su ve yağ depolamak için kullanılan rezervuarlarda büyük hasara yol açabilir. Her yeni roketatarın, yeni aracın tasarımı, yeni yakıt depoları ve karmaşık şekillerdeki rezervuarların üretilmesini gerektirmektedir.

Bu araştırmanın temel amacı, kompozit sıvı dolu rezervuarlardaki titreşimlerin doğal frekanslarını değerlendirmek için sonlu ve sınır elemanlar yöntemlerini entegre ederek güvenilir bir sayısal yaklaşım geliştirmektir. Çalışma özellikle halkalarla birbirine bağlı silindirik ve konik kabuklardan oluşan kabuk yapılarının doğal titreşimlerini analiz etmeye odaklanıyor. Bu kabuklar arasındaki bölge ideal, sıkıştırılamaz bir akışkanla doludur. Sayısal simülasyonlar sınır elemanı tekniklerinin yanı sıra mod süperpozisyon yöntemlerini de kullanır.

Rijit kabuk yapılarındaki sıvı titreşimlerinin spektral sınır problemine sayısal çözüm sunulmuştur. Frekanslar ve modlar, tekil integral denklemlerin çözülmesiyle belirlenir. Dönel kabukların söz konusu olduğu durumlarda bu sistemler, integrallerin eğriler ve doğru parçaları boyunca hesaplandığı tek boyutlu formlara basitleştirilir. Logaritmik ve Cauchy tipi tekilliklere sahip tek boyutlu integralleri hesaplamak için verimli sayısal prosedürler kullanılır.

Sıvı dolu kabukların frekansları ve modları, tekil integral denklem sistemlerinin çözülmesiyle elde edilen temel fonksiyonlarla belirlenir. Test hesaplamaları, önerilen yöntemin yüksek hassasiyetini ve verimliliğini doğrulamaktadır. Bu bulguların önemi ve pratik faydası, farklı uçuş koşulları ve farklı yükler altında farklı yakıt tanklarının titreşimlerini araştırma yeteneğinde yatmaktadır. Ek amaç, söz konusu sistemin mekanik özelliklerini geliştirmek için nanomateryallerin potansiyel kullanımını araştırmaktır. Bu, hesaplamalı modeller geliştirmeyi ve güç, kararlılık ve diğer mekanik özelliklerdeki değişiklikleri değerlendirmek için deneyler yapmayı içerebilir.

Anahtar kelimeler: Koaksiyel kabuklar, Sınır eleman yöntemleri, Çalkantı

ABSTRACT

The issue of fluid oscillations poses a challenge for various industrial sectors, including aerospace, maritime, civil mechanical engineering, nuclear engineering, and is a complex task for physicists and mathematicians as well. Fluid oscillations can lead to catastrophic damage of reservoirs used for storing water and oil. The design of each new rocket launcher, new vehicles necessitate producing new fuel tanks, and reservoirs with intricate shapes.

The main aim of this research is to develop a reliable numerical approach by coupling finite and boundary element methods for evaluating the natural frequencies of vibrations in compound liquid-filled reservoirs. The study specifically focuses on analysing the inherent vibrations of shell structures, comprising interconnected cylindrical and conical shells with rings. The region between these shells is filled with an ideal, incompressible fluid.

The numerical simulations utilize mode superposition methods, along with boundary element techniques. Numerical solution is presented for the spectral boundary problem of liquid vibrations in rigid shell structures. The frequencies and modes are determined by solving singular integral equations. In situations involving shells of revolution, these systems are simplified to one-dimensional forms, where integrals are computed along curves and line segments. Efficient numerical procedures are employed to compute one-dimensional integrals with logarithmic and Cauchy-type singularities. The frequencies and modes of liquid-filled shells are determined through basis functions obtained by solving systems of singular integral equations.

Test calculations validate the high precision and efficiency of the proposed method. The significance and practical utility of these findings lie in the capability to investigate the vibrations of different fuel tanks under diverse flight conditions and different loads. The additional objective is to investigate the potential utilization of nanomaterials for improving the mechanical properties of the system under consideration. This could involve developing computational models and carrying out experiments to evaluate alterations in strength, stability, and other mechanical attributes.

Keywords: Coaxial shells, Boundary element methods, Sloshing

INTRODUCTION

Shells of rotation as fluid storage tanks are widely used in water supply, oil and gas units, in various industries, in nuclear facilities for storage of various liquid or similar materials, chemical liquids and various wastes. Under suddenly applied external loads in partially filled tanks, intensive sloshing in the liquid free surface occurs. Sloshing means any movement of the free surface of the liquid inside the container. This phenomenon has a significant impact on the dynamic response of the container. The spectral boundary value problem for liquid tanks consitsts in finding the natural frequencies and modes of the free surface sloshing.

The calculation of hydrodynamic forces on the walls of reservoirs with liquids is an important problem for ensuring the strength and stability of movement of industrial tanks and vessels. The main problems here are distribution assessment of the hydrodynamic pressure, forces, moments and determination of the natural frequencies of the liquid free surface. These parameters directly affect the dynamic stability and strength of containers. Although only a full-scale experiment can provide an adequate assessment of the strength of tanks with moving liquids, but performing such full-scale experiments is an expensive and dangerous procedure, so the computer modeling is now at the forefront of modern scientific research.

The problem of fluid vibrations is a challenge for various industries, such as aerospace, chemical, mechanical engineering, and nuclear engineering, as well as a challenging task for physicists and mathematicians. Fluctuation of the liquid can lead to catastrophic damage to water and oil storage tanks, descent from the calculated trajectory of launch vehicles. Due to its potentially dangerous effects, liquid sloshing in tanks has been the subject of many theoretical and experimental studies during the last few decades Balas(2023), Gnitko (2016), Lui (2020). Many important phenomena have been addressed in these studies, especially linear and nonlinear slosh effects for both inviscid Gnitko (2019), Strelnikova (2021) and viscous fluids Tong (2020). General overviews of existing solutions to the problem of sloshing are given in Ibragim (2005), Pradeepkumar (2020), Zhang (2021). Note that although compound shells of revolution are typical structures of fuel tanks of launch vehicles and tanks used in the automotive, chemical, and agricultural industries, insufficient attention has been paid to the study of the oscillations of such tanks in the scientific literature Murawski (2020), Karaev (2021), Strelnikva (2019).

Therefore, this study, devoted to the solution of the spectral boundary value problem regarding fluid oscillations in compound shells of revolution, is performed on a topical topic

METHODOLOGY

Research Aim and Problem Formulation

The purpose of the study is to develop computer technology based on the combination of mode superposition and boundary element methods for estimating natural frequencies and modes of fluid oscillations in compound shell structures.

The compound shells of rotation, partially filled with liquid, are considered Fig. 1(a-b). As S_1 the wetted surface of the shell structure is denoted, while S_0 is the free surface of the liquid. It is assumed that the liquid inside shells an is ideal and incompressible one.

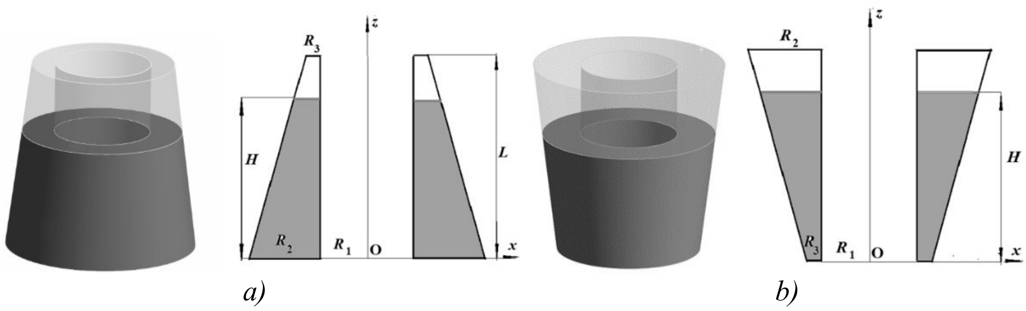

Fig. 1. Compound shell structures and their drafts

The tanks under consideration consist of coaxial cylindrical and conical shells connected by rings that form the bottoms. The liquid is located between the shells. The surfaces of the shells and bottoms are wetted, and the free surface at the state of rest is at a height H, forming a ring, which is described in the polar coordinate system as follows: $\{z=H, R_1<r<R_0\}$.

Here $R_0 = R_3 + (R_2 - R_3)(L - H)/L$ for structure presented at Fig/ 1a), whereas for stucture 1b) it is following $R_0 = R_3 + (R_2 - R_3)H/L$).

Let the movement of the liquid between the shells is vortex-free. Let denote as $\mathbf{V}(V_x, V_y, V_z)$ the liquid velocity vector, then the incompressibility condition takes the form $\mathrm{div}\mathbf{V} = 0$. The condition of the motion potentiality means that there is a scalar velocity potential Φ, such that $\mathbf{V} = \mathrm{grad}\Phi$. It follows from these assumptions that the potential Φ satisfies the Laplace equation in the region occupied by the liquid. Let Ω be an area, occupied by the liquid, **P** is for points inside the liquid domain. The fluid pressure p on the wetted surfaces of the shell is determined from the Bernoulli integral using the formula, Rudnicki (2014)

$$\frac{p}{\rho_l} = -\frac{\partial\Phi}{\partial t} - gz + \frac{p_0}{\rho_l}, \tag{1}$$

where z is the vertical coordinate of the point inside the liquid, g is the gravity acceleration, and ρ_l is the liquid density.

Let formulate the boundary conditions for the Laplace equation for finding the potential Φ. On the wetted surfaces S_1, the non-penetration condition must be fulfilled, while on the free surface S_o dynamic and kinematic conditions are set, Kriutchenko (2017). The non-penetration condition has the form

$$\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_1} = 0. \tag{2}$$

Kinematic and dynamic conditions are fulfilled on the free surface, so

$$\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad p - p_0|_{S_0} = 0, p - p_0 = -\rho\left(\frac{\partial\Phi}{\partial t} + g\zeta\right). \tag{3}$$

Here $\mathbf{n}$ is the unit outward normal to the surface, $\zeta = \zeta(x, y, t)$ is an unknown function that describes the free surface location and its changing in time.

So, the boundary value problem is formulated for the Laplace equation

$$\nabla^2\Phi = 0, \mathbf{P}\in\Omega, \frac{\partial\Phi}{\partial\mathbf{n}} = 0, \ \mathbf{P}\in S_1, \ \frac{\partial\Phi}{\partial\mathbf{n}} = \frac{\partial\zeta}{\partial t}, p - p_0 = 0, \ \mathbf{P}\in S_0 \tag{4}$$

to define the velocity potential Φ with the unknown function $\zeta(x, y, t)$.

Mode superposition method

Since the shells of revolution are considered, and due to the linearity of relations (4), the unknown functions Φ and ζ in cylindrical coordinates are represented as the following series:

$$\zeta(r, \theta, t) = \sum_{l=0}^{m} \cos(l\theta) \sum_{k=1}^{n} d_{kl}(t)\zeta_k(r), \tag{5}$$

$$\Phi(r, \theta, z, t) = \sum_{l=0}^{m} \cos(l\theta) \sum_{k=1}^{n_2} \dot{d}_{kl}(t)\varphi_k(r, z) \tag{6}$$

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At that, the kinematic condition will be fulfilled if the following relationship is valid on the free surface considereing the basic functions $\varphi_k(r,z)$ and $\zeta_k(r)$

$$\left.\frac{\partial\varphi_k(r,z)}{\partial\mathbf{n}}\right|_{z=H} = \zeta_k(r). \tag{7}$$

It follows from the dynamic and kinematic conditions that

$$\frac{\partial\Phi}{\partial t} = -g\zeta, \quad \frac{\partial^2\Phi}{\partial t^2} = -g\frac{\partial\zeta}{\partial t} = -g\frac{\partial\Phi}{\partial\mathbf{n}}. \tag{8}$$

Assuming the harmonic nature of the change in coefficients $d_{kl}(t)$ in time as following $d_{kl}(t) = D_{kl}\exp(i\omega_{kl}t)$, we obtain the next relation:

$$\frac{\partial\psi_{kl}}{\partial\mathbf{n}} = \frac{\omega_{kl}^2}{g}\psi_{kl}, \ \psi_{kl} = \cos(l\theta)\varphi_k(r,z). \tag{9}$$

This leads to the following generalized boundary value problem:

$$\nabla^2\psi_{kl} = 0, \mathbf{P}\in\Omega, \frac{\partial\psi_{kl}}{\partial\mathbf{n}} = 0, \ \mathbf{P}\in S_1, \ \frac{\partial\psi_{kl}}{\partial\mathbf{n}} = \frac{\omega_{kl}^2}{g}\psi_{kl}, \ \mathbf{P}\in S_0. \tag{10}$$

Problem (10) represents the so-called spectral boundry value problem, Raynovskyy (2020).

The boundary element method Brebbia (2017), has been used to solve spectral boundary value problem (10).

Boundary element method

To apply the boundary element method in a direct formulation, Green's third formula is used, Brebbia (2017)

$$2\pi\psi_{kl}(\mathbf{P}_0) = \iint_S \frac{\partial\psi_{kl}}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS - \iint_S \psi_{kl}\frac{\partial}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS, \tag{12}$$

where $|\mathbf{P}-\mathbf{P}_0|$ is the Cartezian distance between points $\mathbf{P}$ and $\mathbf{P}_0$, $S = S_1 \cup S_0$. For spectral problem (10) it follows

$$2\pi\psi_{kl} + \iint_{S_1}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1 - \frac{\omega_{kl}^2}{g}\iint_{S_0}\frac{\psi_{kl}}{|\mathbf{P}-\mathbf{P}_0|}dS_0 + \iint_{S_0}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_0 = 0, \quad \mathbf{P}_0\in S_1$$

$$2\pi\psi_{kl} + \iint_{S_1}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1 + \frac{\omega_{kl}^2}{g}\iint_{S_0}\frac{\psi_{kl}}{|\mathbf{P}-\mathbf{P}_0|}dS_0 = 0, \mathbf{P}_0\in S_0, \tag{13}$$

For shells of revolutopn, using $\psi_{kl} = \cos(l\theta)\varphi_k(r,z)$, one can obtain the followimg system of one-dimensional singular integral equtions:

$$2\pi\varphi_{kl}(r_0,z_0) + \int_\Gamma\varphi_{kl}(r(z),z)\Theta(z,z_0)r(z)d\Gamma - \frac{\omega_{kl}^2}{g}\int_0^R\varphi_{kl}(\rho,H)\Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0, \mathbf{P}_0 \in S_1, \tag{14}$$

$$2\pi\varphi_{kl}(r_0,H) + \int_\Gamma\varphi_{kl}(r(z),z)\Theta(z,z_0)r(z)d\Gamma - \frac{\omega_{kl}^2}{g}\int_0^R\varphi_{kl}(\rho,H)\,\Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0, \mathbf{P}_0 \in S_0,$$

$$\Theta(z,z_0) = \frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2-r_0^2+(z_0-z)^2}{a-b}E_l(k) - F_l(k)\right]n_r + \frac{z_0-z}{a-b}E_l(k)n_z\right\},$$

$$\Xi(P,P_0)=\frac{4}{\sqrt{a+b}}F_l(k),\ a=r^2+r_0^2+(z-z_0)^2, b=2rr_0.$$

Here the generalized elliptical integrals are introduced

$$E_l(k)=(-1)^l(1-4l^2)\int_0^{\pi/2}\cos 2\,b_1 l\theta\sqrt{1-k^2\sin^2\theta}\,d\theta, \tag{15}$$

$$F_l(k)=(-1)^l\int_0^{\pi/2}\frac{\cos 2b_1 l\theta d\theta}{\sqrt{1-k^2\sin^2\theta}}d\theta,\quad k^2=2b/(a+b). \tag{16}$$

For numerical evaluation of integrals (15)-(16) the effective method Karaiev (2021) is applied, that based on the use of an arithmetic-geometric mean Cox (1984). To solve the system of singular equations (14), the method of boundary elements (BEM) with a constant approximation of the density along boundary elements is used, Naumenko (2002), Kriutchenko (2017).

RESULTS AND DISCUSSION

Validation

To test the proposed method, the obtained numerical results are compared with the data Gavrilyuk (2008). Both V-shaped and Λ-shaped conical tanks with R_1 = 1m and $\alpha = \pi/3$ filled eith the liquid are considered. We assume that R_2 is the smaller radius of the cone.

The first frequencies with wave numbers l= 0, 1, 2 are considered, since they are the lowest natural frequencies that determine the hydrodynamic load. The results of the comparison are given in Table 1 for different values of R_2. At the numerical calculations, 120 boundary elements were chosen along the conical part, 100 elements along the free surface radius, and 100 elements along the bottom radius. A further increase in the number of elements did not significantly change the results.

The results obtained by the proposed one-dimensional BEM are in good agreement with the data Gavrilyuk (2008). Some data discrepancy is observed at R_2=0.2m for Λ-shaped conical tanks. At the same time, Gavrilyuk (2008) noted the relatively low accuracy of the semi-analytical method in this particular case.

So, exactly 120 boundary elements are used along both cylindrical and conical surfaces to investigate fluid oscillations in coaxial shells.

Table 1. Sloshing Frequencies in Conical Shells

Method	**V-cone**					**Λ-cone**				
R_2,m	0.2	0.4	0.6	0.8	0.9	0.2	0.4	0.6	0.8	0.9
$l=0$, $k=1$										
Gavrilyuk	3.386	3.386	3.382	3.139	2.187	24.153	10.014	6.665	4.550	2.683
BEM	3.389	3.390	3.391	3.192	2.200	20.027	10.034	6.669	4.545	2.678
$l=1$, $k=1$										
Gavrilyuk	1.304	1.302	1.254	0.934	0.542	11.332	5.629	3.515	1.661	0.726
BEM	1.305	1.307	1.259	0.954	0.574	11.303	5.626	3.481	1.651	0.732
$l=2$, $k=1$										
Gavrilyuk	2.263	2.263	2.255	2.015	1.361	17.760	8.967	5.941	3.724	1.923
BEM	2.265	2.270	2.269	2.048	1.394	17.939	8.965	5.941	3.726	1.951

Sloshing in compound shells

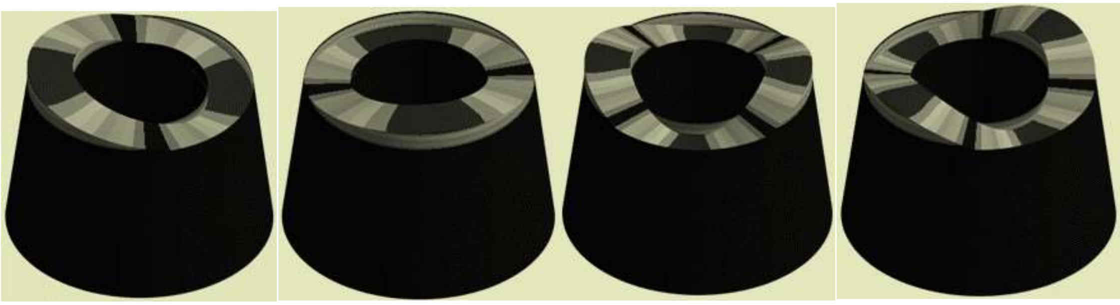
Next, spectral boundary value problem (10) is solved, which made it possible to find the modes $\varphi_k(r,\theta,z)$ and their fundamental frequencies for the structures shown in Fig. 1.

The specificity of these structures is that the free surface has the shape of a ring. The free surface has the same shape when considering toroidal shells. The liquid sloshing in such shells was studied in Karaev (2020). The values of the lower eight sloshing frequencies of coaxial cylindrical-conical shell structures are given in Table 2.

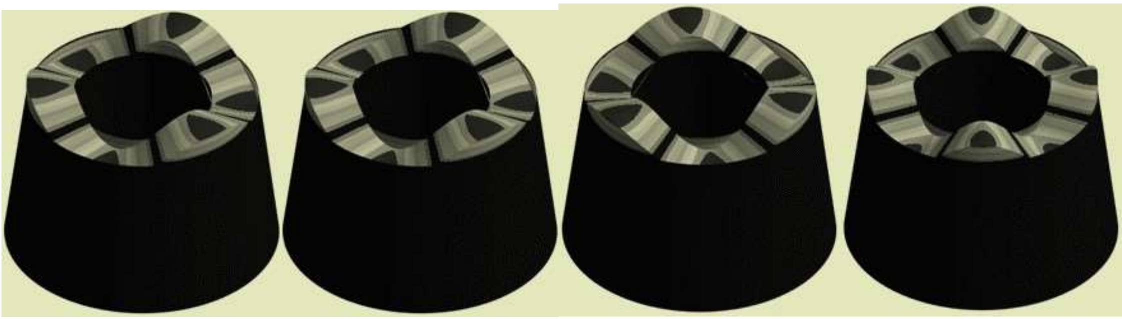
Table 2. Liquid Sloshing Frequeincies in Compound Shells, Hz

№	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Shell structure, Fig.1a)								
frequency	0,6277	0,6277	0,8892	0,8892	1,0779	1,0779	1,2355	1,2355
Shell structure, Fig, 1b)								
frequency	0,5512	0,5512	0,8153	0,8153	1,0027	1,0027	1,1548	1,1548

Note that there are multiple sloshing frequencies. The corresponding sloshing modes of the free surface are shown in Fig. 2 and 3. The sloshing frequencies of both structures differ slightly, but for structures with smaller radius of the free surface, they are higher. This difference decreases with increasing wave number. The lowest frequencies correspond to the first, second and third wave numbers. This corresponds to the calculation data given, regarding sloshing of liquid in conical and cylindrical shells in Ibragim (2005).



1 2 3 4



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5 6 7 8

Fig. 2. Sloshing Modes of Structure, Fig. 1a).

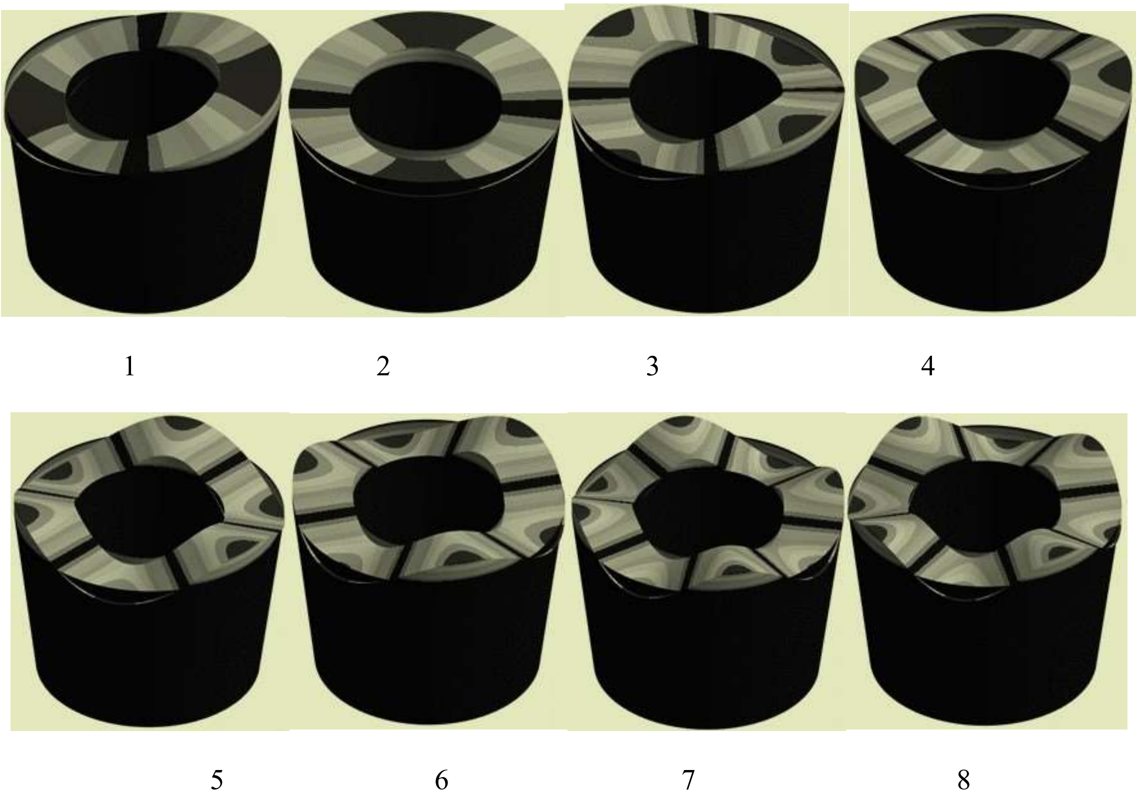

Fig.3. Sloshing Modes of Structure, Fig. 1b)

Thus, the spectral problem of determining the frequencies and modes of fluid vibrations in coaxial shell structures has been solved. This makes it possible to study the movement of liquid in fuel tanks and reservoirs under the action of external loads.

CONCLUSION AND FURTHER RESEARCH

The effaective numerical method for determining the frequencies and modes of fluid oscillations in compound shells of revolution is proposed. The boundary element method is applied, for the first time, to solve the spectral boundary value problem in compound shells of revolution. This approach will be used in computer modeling the dynamic behavior of liquid-filled tanks and the stability study of liquid movement in fuel tanks of complex-shaped launch vehicles.

In the future, it is planned to study the vibrations of elastic coaxial shells with liquid, using various composite materials, Sierikova (2021). Exploring hydroelastic vibrations of conical shells involves considering the interaction of the liquid with the shell walls, including the influence of liquid pressure, its distribution across the shell's surface, and potential resonance phenomena. Modelling with the use of nanomaterials involves studying the potential improvement of mechanical properties of shells, using nanomaterials. This may include numerical models and experiments to assess changes in strength, stability, and other mechanical characteristics.

ACKNOWLEDGMENT

The authors gratefully acknowledge professors Alexander Cheng, Wessex Institute of Technology, and Xiao-Wei Gao, Dalian University of Technology, for their constant supporting of our research.

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