

**Olena Sierikova**

Editor

# **The Fundamentals of Boundary Element Methods**



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## **Chapter 1**

# **Boundary Element Methods for Liquid Hydrocarbon Reservoirs' Vibration Analysis**

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### **Abstract**

Flammable and combustible liquids accumulation on a relatively small area of the tank park leads to an increased environmental and fire hazard of such productions. The possible dangerous liquid leakage and depressurization of tanks negatively affects the surrounding area environment state. A fire in a tank is one of the most dangerous emergency situations that could lead not only to significant material and environmental damage, but also to human casualties. The situation is also complicated by the economically determined tendency to use larger tanks, which significantly increases the flammable liquids volume per unit area. This, in turn, increases the fire spreading risk to neighboring tanks in the absence of timely localization and elimination of the fire. It is urgent and necessary to improve the researching methods of the materials properties of liquid hydrocarbon tanks and to assess the external factors effects of natural and technogenic origin on these environmentally dangerous objects. The determining method of the dynamic characteristics of shell structures made of steel and partially filled with liquid (petroleum products) has been developed in the paper. The dynamic characteristics numerical analysis for the model of liquid hydrocarbons storage tanks has been carried out. The shell structures optimal parameters to reduce their deformations during fluid oscillations (using the liquid hydrocarbon example) have been determined. It has

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been proved that taking into account the tank walls elasticity leads to the significant decrease in the oscillation frequencies compared to the unfilled shells frequencies, while the lowest frequencies of filled and unfilled shell structures could correspond to different wave numbers. Extending the tanks service life, preserving their tightness and stability under various natural and technogenic influences, preventing leaks, spills and fire hazards is necessary to increase the environmental safety level of the surrounding areas.

**Keywords:** boundary element methods, liquid hydrocarbon reservoirs, vibration analysis, environmental safety

## References

- Ablieieva I., Plyatsuk L., Berezhna I., Malovanyy M. Biotechnological Reclamation of Oil-Polluted Soils. *Ecological Engineering & Environmental Technology* (2021) Vol. 22, Issue 2: 27–38.
- Ablieieva I., Plyatsuk L., Roi I., Chekh O., Gabbassova S., Zaitseva K., Lutsenko S. Study of the oil geopermeation patterns: a case study of ANSYS CFX software application for computer modeling. *Journal of Environmental Management* (2021) Vol. 287: 112–347.
- Abramov Y., Basmanov O., Salamov J., Mikhayluk A. Model of thermal effect of fire within a dike on the oil tank. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu* (2018) 2:95–100.
- Abramov Y., Basmanov O., Salamov J., Mikhayluk A., Yashchenko O. Developing a model of tank cooling by water jets from hydraulic monitors under conditions of fire. *Eastern-European Journal of Enterprise Technologies* (2019) 1 (10–97):14–20.
- Akinwumi I. I., Diwa, D., Obianigwe, N. Effects of crudeoil contamination on the index properties, strength andpermeability of lateritic clay. *International Journal of Applied Sciences and Engineering Research* (2014) 3(4): 1-10.
- Akpokodje O. I., Juwah, H. O., Uguru, H. Impacts of petroleum spills on geotechnical propertiesof soils: A review. *Journal of Engineering Innovations and Applications*. (2022) Vol. 1(1): 1-6. <https://doi.org/10.31248/JEIA2022.021>.
- Akpomrere O. R., Uguru, H. Spatial distribution ofresidual petroleum hydrocarbons in an oil spill site located atIsoko South LGA, Delta State, Nigeria. *Journal of Environmentand Waste Management* (2020) 7(1): 312-317.
- Alfach M. T., Wilkinson S. Effect of crude-oil-contaminated soil on the geotechnical behaviour of pilesfoundation. *Geotechnical Research* (2020) 7(2): 76-89.
- Al-Obaidy N., Al-Shueli A., Sattar H., Majeed Z., Hamid N. A. H. An experimental study on geotechnical andelectrical properties of an oil-contaminated soil at thi-qargovernorate/Iraq. *International Review of Civil Engineering*. (2019) 10(3): 148–154.
- Amir Siddiqa M. Deformation and failure in nanomaterials via a data driven modelling approach. *Theoretical and Applied Mechanics Letters* (2020) 10 (4):249-252.

- Avramov K. V., Strel'nikova E. A., Pierre C, Resonant many-mode periodic and chaotic self-sustained aeroelastic vibrations of cantilever plates with geometrical nonlinearities in incompressible flow. *Nonlinear Dyn* (2012) 70:1335–1354 doi:<https://doi.org/10.1007/s11071-012-0537-5>.
- Bouraou N. I., Lukianchenko O., Tsybulnik S., Shevchuk D. Vibration condition monitoring of the vertical steel tanks. *Vibration in Physical Systems* (2016) 27:55-60.
- Bouraou N., Rupich S., Lukianchenko O., Kostina O. Monitoring of the Crack Propagation in Welded Joint of the Tank Using Multi-Class Recognition. *Vibrations in Physical Systems* (2018) 29:8.
- Bozlaker A., Buzcu-Güven B., Fraser M. P., Chellam S. Insights into PM10 sources in Houston, Texas: Role of petroleum refineries in enriching lanthanoid metals during episodic emission events. *Atmos. Environ.* (2013) 69: 109–117.
- Chernysh Y., Ablieieva I., Makarenko N., Plyatsuk L., Trunova I., Burla O. Investigation of the directions of using a hybrid composition bioproduct for detoxification of a soil ecosystem contaminated with heavy metals and oil products. *Biodiversity & Environment*. Prešov : University of Presov (2021) Vol. 13, No. 1: 80–94.
- Dadashov I., Loboichenko V., Kireev A. Analysis of the ecological characteristics of environment friendly fire fighting chemicals used in extinguishing oil products. *Pollution Research* (2018) 37(1):63–77.
- Dadashov I. F., Loboichenko V. M., Strelets V. M., Gurbanova M. A., Hajizadeh F. M., Morozov A. I. About the environmental characteristics of fire extinguishing substances used in extinguishing oil and petroleum products *SOCAR Proceedings*, (2020) 5:79-84. doi: 10.5510/OGP20200100426.
- Ewald V., Ochôa P., Groves R., Boller C. Design of a structural health monitoring system for a damage tolerance fuselage component. *Proceedings of the 7<sup>th</sup> International Symposium on NDT in Aerospace*. Bremen (2015):17- 19.
- Ewetola E. A. Effect of Crude Oil Pollution on some Soil Physical Properties *IOSR Journal of Agriculture and Veterinary Science* (IOSR-JAVS) (2013) 6 (3): 14-17.
- Gnitko V., Degtyarev K., Naumenko V. & Strelnikova E., Coupled BEM and FEM Analysis of fluid-structure interaction in dual compartment tanks. *International Journal of Computational Methods and Experimental Measurements* (2018) 6(6):976–988. doi: 10.2495/CMEM-V6-N6-976-988.
- Gnitko V., Marchenko U., Naumenko V., Strelnikova E., Forced vibrations of tanks partially filled with the liquid under seismic load, *WIT Transaction on Modelling and Simulation* (2011) 52:285-296. doi: 10.2495/BE11025.
- Gnitko V., Karaiev A., Degtyariov K., Vierushkin I., Strelnikova E. Singular and hypersingular integral equations in fluid-structure interaction analysis. *WIT Transactions on Engineering Sciences*. (2022). 134: 67-79. doi:10.2495/BE450061
- Gnitko V., Degtyariov K., Karaiev A., Strelnikova E. Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences*. (2019). vol. 126: 189–200. WIT Press: Southampton and Boston. doi: 10.2495/BE420171.

- Gontarovskyi P., Smetankina N., Garmash N., Melezhyk I. Numerical Analysis of Stress-Strain State of Fuel Tanks of Launch Vehicles in 3D Formulation. In: Nechyporuk, M., Pavlikov, V., Krtskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. ICTM 2020. *Lecture Notes in Networks and Systems* Springer, Cham (2021) 188. [https://doi.org/10.1007/978-3-030-66717-7\\_52](https://doi.org/10.1007/978-3-030-66717-7_52).
- Hentati O., Lachhab R., Ayadi M., Ksibi M. Toxicity assessment for petroleum-contaminated soil using terrestrial invertebrates and plant bioassays. *Environ Monit Asses* (2013) 185: 2989–2998.
- Hetal M. P., Trivedi R. Enhancing Hydrocarbon Degrading Property by Assemblage of Effective Microbial Consortium by Potential Degraders Isolated from Polluted Site. *Applied Ecology and Environmental Sciences*. (2016) 4(2): 44–47. doi: 10.12691/aees-4-2-2.
- Hewelke E., Szatyłowicz J., Hewelke P. et al. The Impact of Diesel Oil Pollution on the Hydrophobicity and CO<sub>2</sub> Efflux of Forest Soils Water Air Soil Pollut. (2018) 229 (51) doi: 10.1007/s11270-018-3720-6.
- Kalabokas P. D., Hatzianestis J., Bartzis J. G., Papagiannakopoulos P. Atmospheric concentrations of saturated and aromatic hydrocarbons around a Greek oil refinery. *Atmos. Environ.* (2001) 35: 2545–2555.
- Karaiev A., Strelnikova E., Singular integrals in axisymmetric problems of elastostatics. *International Journal of Modeling, Simulation, and Scientific Computing* (2020) 11(1): 2050003. doi:10.1142/S1793962320500038.
- Kovalov A., Otrosh Y., Rybka E., Kovalevska T., Togobotska V., Rolin I. Treatment of Determination Method for Strength Characteristics of Reinforcing Steel by Using Thread Cutting Method after Temperature Influence. In *Materials Science Forum*. Trans Tech Publications Ltd. (2020) 1006:179–184.
- Lampart P., Gardzilewicz A., Rusanov A., Yershov S. The effect of stator blade compound lean and compound twist on flow characteristics of a turbine stage - Numerical study based on 3D NS simulations. American Society of Mechanical Engineers. Pressure Vessels and Piping Division (Publication) PVP. (1999). 397 II: 195–204.
- Lukianchenko O. O., Bouraou N. I., Kostina O. V., Kuzko O. V. Investigation of Static and Dynamic Characteristics of Complex Thin-Walled Shell Structure with Cracks. *Strength of Materials* (2016) 48(3):401– 410.
- Maliszewska-Kordybach Barbara, Suszek-Łopatka Beata, Klimkowicz-Pawlas Agnieszka, Smreczak Bożena. Influence of temperature on phenanthrene toxicity towards nitrifying bacteria in three soils with different properties. *Environmental Pollution*. (2016) Vol. 216: 911–918. <https://doi.org/10.1016/j.envpol.2016.06.066>.
- Marín-García D C, Adams R. H., Hernández-Barajas R. Effect of crude petroleum on water repellency in a clayey alluvial soil. *International journal of Environmental Science and Technology* (2016) 13 (1): 55–64.
- Misura, S., Smetankina, N., Misiura, i.e.: *Optimal design of the cyclically symmetrical structure under static load*. In: Nechyporuk, M., Pavlikov, V., Krtskiy, D. (eds.) ICTM 2020. LNNS, 188: 256–266. Springer, Cham (2021). [https://doi.org/10.1007/978-3-030-66717-7\\_21](https://doi.org/10.1007/978-3-030-66717-7_21).

- Moubasher H. A., Hegazy A. K., Mohamed N. H., Moustafa Y. M., Kabil H. F., Hamad A. A. Phytoremediation of soils polluted with crude petroleum oil using Bassia scoparia and its associated rhizosphere microorganisms. *International Biodeterioration & Biodegradation*. (2015) 98: 113-120.
- Nasehi S. A., Uromiehy A., Nikudel M. R., Morsali A. Influence of gas oil contamination on geotechnical properties of fine and coarse-grained soils. *Geotechnical and Geological Engineering*. (2016) 34(1): 333-345.
- Ngene S., Tota-Maharaj K., Eke P., Hills C. Environmental and Economic Impacts of Crude Oil and Natural Gas Production in Developing Countries International Journal of Economy. *Energy and Environment* (2016) 1(3): 64-73 doi: 10.11648/j.ijeee.20160103.13.
- Ostovar M., Ghiasi R., Mehdizadeh M. J., Shariatmadari N. Effects of crude oil on geotechnical specification of sandy soils. *Soil and Sediment Contamination: An International Journal*. (2020) 30(1): 58-73.
- Oyedele A., Adebiyi M. Omotayo C. Ogunkunle. Effect of Crude Oil-Contaminated Soil on Germination and Growth Performance of Abelmoschus esculentus L. Moench—A Widely Cultivated Vegetable Crop in Nigeria. *American Journal of Plant Sciences*. (2012) Vol. 3 No. 10: 1451-1454. doi: 10.4236/ajps.2012.310174.
- Rabinovich S. Measurement errors and uncertainties theory and practice. Third edition. Springer Science and Media, Inc. USA (2005):308.
- Ragothaman A., Anderson W. A. Air Quality Impacts of Petroleum Refining and Petrochemical Industries. *Environments* (2017) 4, 66. <https://doi.org/10.3390/environments4030066>.
- Romic L., vel-Cerovecki S. Atmospheric Emissions from Sources of Air Pollution in Petroleum Industry-Emission Inventory. *SPE International Conference on Health, Safety and Environment in Oil and Gas Exploration and Production*, Stavanger, Norway, June (2000). doi: <https://doi.org/10.2118/61509-MS>.
- Rusanov, A., Shubenko, A., Senetskyi, O., Babenko, O., Rusanov, R. Heating modes and design optimization of cogeneration steam turbines of powerful units of combined heat and power plant | Didelių kombinuoto ciklo jėgainių kogeneracinių garo turbinų šildymo režimai ir dizaino optimizavimas. *Energetika*, 2019, 65(1), 39–50.
- Rusanov A. V., Solovey V. V. and Lototskyy M. V., Thermodynamic features of metal hydride thermal sorption compressors and perspectives of their application in hydrogen liquefaction systems. *Journal of Physics: Energy* (2020) 2(2): 021007 DOI 10.1088/2515-7655/ab7bf4.
- Rusanov A. V., Kostikov A. O., Shubenko O. L., Kharlampidi D. K., Tarasova V. A., Senetskyi O. V., Highly efficient cogeneration power plant with deep regeneration based on air Brayton cycle. *J Mech Eng.* (2019) 22(1):2–23. <https://doi.org/10.15407/pmach2019.04.012>.
- Semerak M., Pozdeev S., Yakovchuk R., Nekora O., Sviatkevych O. Mathematical modeling of thermal fire effect on tanks with oil products. *MATEC Web of Conferences*. (2018) 247. doi.org/10.1051/matecconf/201824700040.

- Shevchenko R. I., Strelets V. M., Loboichenko V. M., Pruskyi A. V., Myroshnyk O. N., Kamyshentsev G. V. Review of up-to-date approaches for extinguishing oil and petroleum products *SOCAR Proceedings* (2021):169-174. doi: 10.5510/OGP2021SI100519.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova E. Improving the Mechanical Properties of Liquid Hydrocarbon Storage Tank Materials. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland (2022) 1068:223-229. doi:10.4028/p-888232.
- Sierikova O., Strelnikova E., Kriutchenko D., Gnitko V. Reducing Environmental Hazards of Prismatic Storage Tanks under Vibrations. *WSEAS Transactions on Circuits and Systems* (2022) 21:249-257. doi: 10.37394/23201.2022.21.27.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova O. The Deformable and Strength Characteristics of Nanocomposites Improving. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland (2021) 1038:144-153. <https://doi.org/10.4028/www.scientific.net/MSF.1038.144>.
- Sierikova O., Strelnikova E., Degtyarev K. Strength. Characteristics of Liquid Storage Tanks with Nanocomposites as Reservoir Materials. *2022 IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek)* (2022):151-157. doi: 10.1109/KhPIWeek57572.2022.9916369.
- Sierikova O., Strelnikova E., Degtyarev K. Seismic Loads Influence Treatment on the Liquid Hydrocarbon Storage Tanks Made of Nanocomposite Materials. *WSEAS Transactions on Applied and Theoretical Mechanics* (2022) 17:62-70. doi: 10.37394/23201.2022.17.9.
- Sierikova O., Strelnikova E., Gnitko V., Degtyarev K. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. *IEEE 2<sup>nd</sup> KhPI Week on Advanced Technology (KhPIWeek)* (2021):351–356. doi: 10.1109/KhPIWeek53812.2021.9570086.
- Sikkema J., de Bont A. M., Poolman B. *Mechanisms of Membrane Toxicity of Hydrocarbons Microbiological REVIEWS*. (1995) Vol. 59, 2: 201-222.
- Shevchyk L., Romaniuk O. The optimal way of biological cleaning of oil-contaminated soils. *Mediterranean Journal of Biosciences*. (2016) 1(3): 109-113. <http://ojs.medjbio.com/index.php/medjbio/article/view/396>.
- Surianinov M., Andronov V., Otrosh Y., Makovkina T., Vasiukov S. Concrete and fiber concrete impact strength. *Materials Science Forum*. (2020) 1006 MSF:101–106.
- Taghehbaf M. A., Givehchi S., Ardestani M., Baghvand A. 2014 Modeling the Consequences of Potential Accidents in One of the Gasoline Storage Tanks at Oil Storage of Yazd, in Terms of *Explosion International Journal of Engineering Innovation & Research* (2014) 3 (4): 555-560.
- Vambol S., Vambol V., Suchikova Y., Deyneko N. Analysis of the ways to provide ecological safety for the products of nanotechnologies throughout their life cycle. *Eastern-European Journal of Enterprise Technologies* (2017) 1(10–85):27–36.
- Van Hamme J. D., Singh A., Ward O. P. *Recent Advances in Petroleum Microbiology. Microbiology and Molecular Biology Reviews*. Dec. 2003, p. 503–549 Vol. 67, No. 4.

Wang Q., Zhang Sh., Li Yu., Klassen W. Potential Approaches to Improving Biodegradation of Hydrocarbons for Bioremediation of Crude Oil Pollution. *Journal of Environmental Protection* (2011): 47-55 doi:10.4236/jep.2011.21005.



## **Chapter 2**

# **Boundary and Finite Element Methods in Nanocomposites Effective Elastic Characteristics Estimation**

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## **Abstract**

The main objective of this research has been to develop competent methods to estimate effective elastic modulus of composites and nanocomposites with randomly arranged cylindrical nanoinclusions with different ratios of fiber length to its thickness. The concept of representative volume elements has been involved. Series of calculations has been carried out for the nanocomposite material consisting of metallic aluminium matrix with inclusions as the carbon nanocylinders.

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As a result of these calculations, elastic properties for the orthotropic composite material have been obtained, namely, Young's modulus, shear modulus, Poisson's ratio. It has been demonstrated that distributions with uniaxial filler orientation in the matrix, the inclusion shapes and properties, have a significant effect on the final elastic, orthotropic composite properties. The proposed models and methods provide an effective tool for predicting the mechanical properties of three-dimensional matrix nanocomposites.

**Keywords:** randomly arranged cylindrical inclusion, nanocomposites, representative volume element, boundary element methods, finite element methods, environmental safety

## References

- Avramov K. V., Strel'nikova E. A., Pierre C. Resonant many-mode periodic and chaotic self-sustained aeroelastic vibrations of cantilever plates with geometrical nonlinearities in incompressible flow. *Nonlinear Dyn.* (2012) vol. 70: 1335–1354. <https://doi.org/10.1007/s11071-012-0537-5>.
- Danchenko Y., Andronov V., Barabash E., Obigenko T., Rybka E., Meleshchenko R., Romin A.. Research of the intramolecular interactions and structure in epoxyamine composites with dispersed oxides. *Eastern-European Journal of Enterprise Technologies.* (2017). vol. 6: 12–90.
- Degtyariov K., Gnitko V., Kononenko Y., Kriutchenko D., Sierikova O., Strelnikova E. Fuzzy Methods for Modelling Earthquake Induced Sloshing in Rigid Reservoirs. 2022 *IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek).* (2022): 297-302. doi: [10.1109/KhPIWeek57572.2022.9916466](https://doi.org/10.1109/KhPIWeek57572.2022.9916466).
- Gnitko V., Degtyarev K., Naumenko V., Strelnikova E. Coupled BEM and FEM Analysis of fluid-structure interaction in dual compartment tanks, *International Journal of Computational Methods and Experimental Measurements.* (2018). 6(6): 976–988. doi: [10.2495/CMEM-V6-N6-976-988](https://doi.org/10.2495/CMEM-V6-N6-976-988).
- Gnitko, V., Marchenko, U., Naumenko, V., Strelnikova, E., Forced vibrations of tanks partially filled with the liquid under seismic load. *WIT Transaction on Modelling and Simulation.* (2011). vol. 52: 285-296. doi: [10.2495/BE11025](https://doi.org/10.2495/BE11025).
- Gnitko V., Degtyariov K., Karaiev A., Strelnikova E. Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences.* (2019). vol. 126: 189–200. WIT Press: Southampton and Boston. doi: [10.2495/BE420171](https://doi.org/10.2495/BE420171).
- Gurtin M.E., Murdoch A.I. A continuum theory of elastic material surfaces. *Archive of Rational Mechanics Analysis.* (1975). vol. 57: 291–323.
- Karaiev A., Strelnikova E. Singular integrals in axisymmetric problems of elastostatics, *International Journal of Modeling, Simulation, and Scientific Computing.* (2020). vol. 11 (1): 200003. doi: [10.1142/S1793962320500038](https://doi.org/10.1142/S1793962320500038).

- Kovalov A., Otrosh Y., Rybka E., Kovalevska T., Togobyska V., Rolin I. Treatment of Determination Method for Strength Characteristics of Reinforcing Steel by Using Thread Cutting Method after Temperature Influence. In *Materials Science Forum*. Trans Tech Publications Ltd. (2020). Vol. 1006: 179-184.
- Kurenkov S., Smetankina N., Pavlikov V., Dvoretskaya D., Radchenko V. Mathematical Model of the Stress State of the Antenna Radome Joint with the Load-Bearing Edging of the Skin Cutout. In: Ciobătană, D. D. (eds) International Conference on Reliable Systems Engineering (ICoRSE). 2021. ICoRSE 2021. *Lecture Notes in Networks and Systems*. (2021). vol 305. Springer, Cham. [https://doi.org/10.1007/978-3-030-83368-8\\_28](https://doi.org/10.1007/978-3-030-83368-8_28).
- Kushch V. I. Atomistic and continuum modeling of nanoparticles: Elastic fields, surface constants, and effective stiffness. *International Journal of Engineering Science*. (2023). vol. 183: 103806. doi.org/10.1016/j.ijengsci.2022.103806.
- Kushch V. I., “Stress field and effective elastic moduli of periodic spheroidal particle composite with Gurin-Murdoch interface. *International Journal of Engineering Science*. (2018). vol. 132: 79-96. doi: 10.1016/j.ijengsci. 2018.08.00.
- Le M. T., Huang S. C. Modeling and Estimating the Effective Elastic Properties of Carbon Nanotube Reinforced Composites by Finite Element Method. *J. Eng. Technol. Educ.* (2014). 11(2): 145-158.
- Miller R. E., Shenoy V. B., Size-dependent elastic properties of nanosized structural elements. *Nanotechnology*. (2000). 11: 139-147.
- Misura S., Smetankina N., Misiura Ie. Optimal design of the cyclically symmetrical structure under static load. In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds.) *ICTM 2020*. (2021). LNNS, 188: 256–266. Springer, Cham. [https://doi.org/10.1007/978-3-030-66717-7\\_21](https://doi.org/10.1007/978-3-030-66717-7_21).
- Mykhas'kiv V. V., Stasyuk B. M. Effective elastic properties of 3D composites with short curvilinear fibers: numerical simulation and experimental validation. *Solid State Phenomena*. (2017). vol. 258: 452-455.
- Rusanov A. V., Solovey V. V., Lototskyy M. V. Thermodynamic features of metal hydride thermal sorption compressors and perspectives of their application in hydrogen liquefaction systems. *Journal of Physics: Energy*. (2020). vol. 2(2): 021007. doi: 10.1088/2515-7655/ab7bf4.
- Shugaylo O., Bilyk S. The Specifics of the Compilation of the Calculated Load Combinations in the Assessment of Seismic Resistance of Steel Supporting Structures of Nuclear Power Plant Equipment and Piping. *J. of Mech. Eng.* (2022). vol. 25(3): 6-15. <https://doi.org/10.15407/pmach2022.03.006>.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova E. Improving the Mechanical Properties of Liquid Hydrocarbon Storage Tank Materials. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland. (2022). vol. 1068: 223-229. doi:10.4028/p-888232.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova O. The Deformable and Strength Characteristics of Nanocomposites Improving. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland. (2021). vol. 1038: 144-153. doi: 10.4028/www.scientific.net/MSF.1038.144.

- Sierikova O., Koloskov V., Strelnikova E. The groundwater level changing processes modeling in 2d and 3d formulation. *Acta Periodica Technologica*. (2022). 53: 36-47. doi: <https://doi.org/10.2298/APT2253036S>.
- Sierikova O., Strelnikova E., Degtyarev K. Seismic Loads Influence Treatment on the Liquid Hydrocarbon Storage Tanks Made of Nanocomposite Materials. *WSEAS Transactions on Applied and Theoretical Mechanics*. (2022). vol. 17: 62-70. doi: 10.37394/232011.2022.17.9.
- Sierikova O., Strelnikova E., Degtyarev K. Strength Characteristics of Liquid Storage Tanks with Nanocomposites as Reservoir Materials. *2022 IEEE 3<sup>rd</sup> KhPI Week on Advanced Technology (KhPIWeek)*. (2022): 151-157. doi: [10.1109/KhPIWeek57572.2022.9916369](https://doi.org/10.1109/KhPIWeek57572.2022.9916369).
- Sierikova O., Strelnikova E., Gnitko V., Degtyarev K. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. *2021 IEEE 2<sup>nd</sup> KhPI Week on Advanced Technology (KhPIWeek)*. (2021): 351-356. doi: 10.1109/KhPI Week53812.2021.9570086.
- Sierikova O., Strelnikova E., Gnitko V., Tonkonozhenko A., Pisnia L. Nanocomposites Implementation for Oil Storage Systems Electrostatic Protection. *Conf. Proc. of Integrated Computer Technologies in Mechanical Engineering ICTM-2021*. Synergetic Engineering Springer Nature Switzerland AG 2022 M. Nechyporuk et al. (Eds.): ICTM 2021. (2022). LNNS 367: 573-585. doi.org/10.1007/978-3-030-94259-5\_49.
- Sierikova O., Strelnikova E., Kriutchenko D. Membrane installation in storage tanks for seismic loads impact protection. *Acta Periodica Technologica*. (2023). (54): 209-222. <https://doi.org/10.2298/APT2354209S>.
- Sierikova O., Strelnikova E., Kriutchenko D., Gnitko V. Reducing Environmental Hazards of Prismatic Storage Tanks under Vibrations. *WSEAS Transactions on Circuits and Systems*. (2022). vol. 21: 249-257. doi: 10.37394/23201.2022.21.27.
- Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E. Flood risk management of Urban Territories. *Ecology, Environment and Conservation*. (2020). vol. 26 (3): 1068-1077.
- Smetankina N. V., Shupikov A. N., Sotrikhin S. Yu., Yareschenko V. G. A noncanonically shape laminated plate subjected to impact loading: Theory and experiment. *Journal of Applied Mechanics, Transactions ASME*. (2008). vol. 75(5): 051004-1–051004-9. <https://doi.org/10.1115/1.2936925>.
- Surianinov M., Andronov V., Otrosh Y., Makovkina T., Vasiukov S. Concrete and fiber concrete impact strength. *Materials Science Forum*. (2020). 1006 MSF: 101-106. doi.org/10.4028/www.scientific.net/MSF.1006.101.
- Tulskyi H. H., Liashok L. V., Shevchenko H. S., Vasilchenko A. V., Stelmakh O. A. Synthesis of functional nanocomposites based on aluminum oxide. *Functional Materials*. (2019). vol. 26 (4): 718-722. doi.org/10.15407/fm26.04.718.
- Vambol S., Vambol V., Suchikova Y., Deyneko N. Analysis of the ways to provide ecological safety for the products of nanotechnologies throughout their life cycle. *Eastern European Journal of Enterprise Technologies*. (2017). 1/10(85): 27-36. DOI: <https://doi.org/10.15587/1729-4061.2017.85847>.

Vambol S., Bohdanov I., Vambol V. Suchikova Y., Kondratenko O., Nestorenko T., Onyschenko S. Formation of filamentary structures of oxide on the surface of monocrystalline gallium arsenide. *Journal of Nano- and Electronic Physics.* (2017). vol. 9(6) № 06016. DOI: 10.21272/jnep.9(6).06016.

Zheng J., Xue M., Dou P., He Y. A review on liquid sloshing hydrodynamics. *Journal of Hydrodynamics.* (2021). vol. 33(6): 1089-1104. doi.org/10.1007/s42241-022-0111-7.

## **Chapter 3**

# **Boundary Element Methods in One-Dimensional Singular Integral Equations**

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### **Abstract**

The aim of this paper is to analyze numerical estimations effectiveness in one-dimensional singular integral equations implementing boundary element methods. The types of elements have been investigated. Especially attention has been paid out to choose collocation points. The constant, linear, quadratic, and cubic approximations of densities along the boundary elements have been considered, and the effectiveness of these approximations has been estimated. The optimal positions of the collocation points have been obtained for accurate calculations of the singular integrals. The obtained numerical results show that even with a small number of linear elements, they give good accuracy, but it couldn't be improved for  $n < 100$ . The convergence rate with quadratic elements changes according to a quadratic law; cubic elements are according to a cubic one. To obtain stable numerical schemes using boundary element methods, it is necessary to arrange control points in a special way, namely in maximum value points of responses.

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**Keywords:** one-dimensional boundary elements, collocation points, density approximations

## References

- Ahues M., Mennouni A. A Collocation Method for Cauchy Integral Equations in L2. In: Constanda, C., Harris, P. (eds) *Integral Methods in Science and Engineering*. Birkhäuser Boston. (2011). [https://doi.org/10.1007/978-0-8176-8238-5\\_1](https://doi.org/10.1007/978-0-8176-8238-5_1).
- Brebbia C. A., Telles J. C. F., Wrobel L. C. *Boundary Element Techniques*. Springer-Verlag: Berlin and New York, 1984.
- Degtyariov K., Gnitko V., Kononenko Y., Kriutchenko D., Sierikova O., Strelnikova E. Fuzzy Methods for Modelling Earthquake Induced Sloshing in Rigid Reservoirs. 2022 *IEEE 3<sup>rd</sup> KhPI Week on Advanced Technology* (KhPIWeek). (2022): 297-302. doi: 10.1109/KhPIWeek57572.2022.9916466.
- Gnitko, V., Degtyariov, K., Karaiev, A., & Strelnikova, E.: Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences*. (2019). 126: 189–200. doi: 10.2495/BE420171.
- Gnitko V., Karaiev, A., Degtyariov, K., Vierushkin, I., Strelnikova, E.: Singular and hypersingular integral equations in fluid–structure interaction analysis. *WIT Transactions on Engineering Science*. (2022). 134: 67-79. doi:10.2495/BE450061.
- Gnitko V., Marchenko U., Naumenko V., Strelnikova E. Forced vibrations of tanks partially filled with the liquid under seismic load. *WIT Transaction on Modelling and Simulation*. (2011). vol. 52: 285-296. doi: 10.2495/BE11025.
- Zaitsev B. P., Protasova, T. V., Smetankina, N. V., Klymenko, D. V. Larionov, I. F. Akimov, D. V.: Oscillations of the Payload Fairing Body of the Cyclone-4M Launch Vehicle during Separation. *Strength Mater.* (2020). 52(6): 849-863. doi:10.1007/s11223-021-00239-5.
- Karaiev A., Strelnikova E. Axisymmetric polyharmonic spline approximation in the dual reciprocity method. *ZAMM-Journal of Applied Mathematics and Mechanics. Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 101, № 1, e201800339. doi:10.1002/zamm.201800339.
- Karaiev A., Strelnikova E. Singular integrals in axisymmetric problems of elastostatics/*International Journal of Modeling. Simulation, and Scientific Computing*. (2020). 11(1): 2050003. doi:10.1142/S1793962320500038.
- Lampart P., Rusanov A., Yershov S., Marcinkowski S., Gardzilewicz, A. Validation of a 3D BANS solver with a state equation of thermally perfect and calorically imperfect gas on a multi-stage low-pressure steam turbine flow. *Journal of Fluids Engineering, Transactions of the ASME*. (2005). 127(1): 83–93.
- Liu Z., Wei G., Qin S., Wang Z. The elastoplastic analysis of functionally graded materials using a meshfree RKPM. *Applied Mathematics and Computation*. (2022). 413: 126651. <https://doi.org/10.1016/j.amc.2021.126651>.
- Muskhelishvili N. *Singular Integral Equations*, 1958. Republished by Springer Netherlands, December (2012). doi:10.1007/978-94-009-9994-7.

- Rizzo F. J., Shippy D. J. An advanced boundary integral equation method to three-dimensional thermo-elasticity. *Int. J. Num. Meth. in Engng.* (1977). Vol. 11, № 17: 1753-1760. doi.org/10.1002/nme.1620111109.
- Rusanov A., Shubenko A., Senetskyi O., Babenko O., Rusanov R. Heating modes and design optimization of cogeneration steam turbines of powerful units of combined heat and power plant. *Energetika.* (2019). vol. 65(1): 39-50. doi.org/10.6001/energetika.v65i1.3974.
- Seifi A. Numerical solution of certain Cauchy singular integral equations using a collocation scheme. *Adv Differ Equ.* (2020). 537. <https://doi.org/10.1186/s13662-020-02996-0>.
- Serikova E., Strelnikova E., Yakovlev V. Mathematical model of dangerous changing the groundwater level in Ukrainian industrial cities. *Journal of Environment Protection and Sustainable Development.* (2015). No 1: 86-90. <https://www.researchgate.net/publication/281784323>.
- Sierikova O., Koloskov V., Strelnikova E. The groundwater level changing processes modeling in 2d and 3d formulation. *Acta Periodica Technologica.* (2022). 53: 36-47. doi: <https://doi.org/10.2298/APT2253036S>.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova O. The Deformable and Strength Characteristics of Nanocomposites Improving. *Materials Science Forum.* Trans Tech Publications, Ltd, Switzerland. (2021). 1038: 144-153. doi.org/10.4028/www.scientific.net/MSF.1038.144.
- Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E. *Flood risk management of Urban Territories.* Ecology, Environment and Conservation, vol. (2020). 26 (3): 1068-1077.
- Sierikova O., Strelnikova E., Gnitko V., Degtyarev K. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. *2021 IEEE 2nd KhPI Week on Advanced Technology (KhPIWeek).* (2021): 351-356. doi: 10.1109/KhPIWeek53812.2021.9570086.
- Smetankina N. V., Shupikov A. N., Sotrikhin S. Yu., Yareschenko, V. G. Dynamic response of an elliptic plate to impact loading. *Theory and experiment International Journal of Impact Engineering.* (2007). 34 (2): 264-276. <https://doi.org/10.1016/j.ijimpeng.2005.07.016>.

## **Chapter 4**

# **Boundary Element Methods for Hypersingular Integral Equations Over Circular Domains**

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### **Abstract**

The purpose of this study is to analyze and systematize existing and obtain new analytical solutions of hypersingular integral equations, clarify the properties of these solutions, and compare analytical and numerical solutions. Circular domains have been chosen as areas of integration. Analytical solutions of these equations have been constructed, and it has been established that the potential densities of the double layer potential repeat the character of the right parts of the corresponding hypersingular equations. The boundary element methods have been applied to numerical simulations. The comparison of numerical and analytical results has been provided. The problems of determining the stress intensity factors in an infinite elastic body have been analyzed using different approaches concerned with hypersingular integral equations as well as with finite element methods. The solutions of hypersingular integral equations in vibration analysis of circle plates have been presented.

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**Keywords:** Laplace's equation, boundary value problems, fluid-structure interaction, hypersingular integral equation, analytical and numerical solutions, stress concentration

## References

- Beirão da Veiga H., Perturbation theory and well-posedness in Hadamard's sense of hyperbolic initial-boundary value problems. *Nonlinear Analysis: Theory, Methods & Applications.* (1994). Vol. 22, Issue 10: 1285-1308. [https://doi.org/10.1016/0362-546X\(94\)90111-2](https://doi.org/10.1016/0362-546X(94)90111-2).
- Brebbia C. A., Telles J. C. F, Wrobel L. C. Boundary element techniques: theory and applications in engineering. *Springer-Verlag*: Berlin and New York. (1984).
- Gnitko V., Karaiev A., Degtyariov K., Vierushkin I., Strelnikova E.: Singular and hypersingular integral equations in fluid-structure interaction analysis. *WIT Transactions on Engineering Sciences.* (2022). 134: 67-79. doi:10.2495/BE450061.
- Gnitko V., Degtyarev K., Naumenko V., Strelnikova E. Coupled BEM and FEM Analysis of fluid-structure interaction in dual compartment tanks. *International Journal of Computational Methods and Experimental Measurements.* (2018). 6(6): 976–988. doi: 10.2495/CMEM-V6-N6-976-988.
- Gnitko V., Degtyariov K., Naumenko V., Strelnikova E. BEM and FEM analysis of the fluid-structure interaction in tanks with baffles. *International Journal of Computational Methods and Experimental Measurements.* Southampton. (2017). 5(3): 317-328. doi:10.2495/CMEM-V5-N3-317-328.
- Karaiev A., Strelnikova E. Axisymmetric polyharmonic spline approximation in the dual reciprocity method. *ZAMM-Journal of Applied Mathematics and Mechanics. Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 101, № 1: e201800339. doi:10.1002/zamm.201800339.
- Karaiev A., Strelnikova E. Singular integrals in axisymmetric problems of elastostatics. *International Journal of Modeling. Simulation, and Scientific Computing.* (2020). 11(1): 2050003. doi:10.1142/S1793962320500038.
- Lifanov I. K. *Singular Integral Equations and Discrete Vortices*, Berlin, Boston, De Gruyter, (1996). doi:10.1115/9783110926040.
- Makeev V. I., Strelnikova E. A., Trofimenco P. E., Bondar A. V. On Choice of Design Parameters for an Aircraft. *Int. Appl. Mech.* (2013). 49, N 5: 588 – 596. doi: 10.1007/s10778-013-0592-8.
- Moskalenko R., Zaydenvarg O., Strelnikova O., Gnitko V. Software development for the computational analysis of crack propagation and durability of structures. *2020 IEEE KhPI Week on Advanced Technology (KhPIWeek).* (2020): 509-514. doi: 10.1109/KhPIWeek51551.2020.9250089.
- Mykhas'kiv V. V., Stasyuk B. M. Effective elastic properties of 3D composites with short curvilinear fibers: numerical simulation and experimental validation. *Solid State Phenomena.* (2017). Vol. 258: 452-455.

- Naumenko V. V., Strelnikova E. A Singular Integral Accuracy of Calculations in Two-Dimensional problems. *Eng. analysis with boundary elements*. (2002). No 26: 95–98. doi: 10.1016 / S0955-7997 (01) 00041-8.
- Obaiys S. J., Ibrahim R. W., Ahmad A. F. Hypersingular Integrals in Integral Equations and Inequalities: Fundamental Review Study. In: *Differential and Integral Inequalities*. Springer Optimization and Its Applications. (2019). 151. //doi.org/10.1007/978-3-030-27407-8\_25.
- Rusanov A. V., Kostikov A. O., Shubenko O. L., Kharlampidi D. K., Tarasova V. A., Senetskyi O. V., Highly efficient cogeneration power plant with deep regeneration based on air Brayton cycle. *J Mech Eng.* (2019). 22:1: 2–23. https://doi.org/10.15407/pmach2019.04.012.
- Rusanov A., Shubenko A., Senetskyi O., Babenko O., Rusanov R. Heating modes and design optimization of cogeneration steam turbines of powerful units of combined heat and power plant. *Energetika*. (2019). vol. 65(1): 39-50. doi.org/10.6001/energetika.v65i1.3974.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova O. The Deformable and Strength Characteristics of Nanocomposites Improving. *Materials Science Forum*. (2021). 1038: 144–153. <https://doi.org/10.4028/www.scientific.net/msf.1038.144>.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova E. Improving the Mechanical Properties of Liquid Hydrocarbon Storage Tank Materials. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland. (2022). Vol. 1068: 223-229. doi:10.4028/p-888232.
- Sierikova O., Strelnikova E., Degtyariov K. Numerical Simulation of Strength and Aerodynamic Characteristics of Small Wind Turbine Blades. In: Nechyporuk, M., Pavlikov, V., Krtskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2022. ICTM 2022. *Lecture Notes in Networks and Systems*. (2023). vol 657. Springer, Cham. [https://doi.org/10.1007/978-3-031-36201-9\\_31](https://doi.org/10.1007/978-3-031-36201-9_31).
- Sierikova O., Strelnikova E., Degtyarev K. Seismic Loads Influence Treatment on the Liquid Hydrocarbon Storage Tanks Made of Nanocomposite Materials. *WSEAS Transactions on Applied and Theoretical Mechanics*. (2022). vol. 17: 62-70. doi: 10.37394/232011.2022.17.9.
- Sierikova O., Strelnikova E., Degtyarev K. Strength Characteristics of Liquid Storage Tanks with Nanocomposites as Reservoir Materials. *2022 IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek)*. (2022): 151-157. doi: [10.1109/KhPIWeek57572.2022.9916369](https://doi.org/10.1109/KhPIWeek57572.2022.9916369).
- Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E. Flood risk management of Urban Territories. *Ecology, Environment and Conservation*. (2020). vol. 26 (3): 1068-1077.
- Serikova E., Strelnikova E., Yakovlev V. Mathematical model of dangerous changing the groundwater level in Ukrainian industrial cities. *Journal of Environment Protection and Sustainable Development*. (2015). vol. 1: 86-90. /Files/journals/JTME/V3No1/StrelnikovaE.pdf.
- Smetankina N. V., Shupikov A. N., Sotrikhin S. Yu., Yareschenko V. G. Dynamic response of an elliptic plate to impact loading. Theory and experiment. *International Journal of Impact Engineering*. (2007). 34 (2): 264-276. https://doi.org/10.1016/j.ijimpeng.2005.07.016.

Zaitsev B. P., Protasova T. V., Smetankina N. V., Klymenko D. V. Larionov I. F. Akimov D. V. Oscillations of the Payload Fairing Body of the Cyclone-4M Launch Vehicle during Separation. *Strength Mater.* (2020). 52(6): 849-863. doi:10.1007/s11223-021-00239-5.

Zhang J., Xu R., He Y., Yang W. Direct Computation of 3-D Stress Intensity Factors of Straight and Curved Planar Cracks with the P-Version Finite Element Method and Contour Integral Method. *Materials.* (2021). 14(14): 3949. doi:10.3390/ma14143949.

## **Chapter 5**

# **Boundary Element Method Testing for Axially-Symmetrical Problems**

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### **Abstract**

The purpose of this study is to develop and test boundary element methods for axially-symmetrical integration domains. The liquid-filled revolution shells' vibrations have been considered in coupled formulation. The shell motion equation in the absence of external perturbations has been described on the basis of the Ostrogradsky–Hamilton principle. To determine the fluid pressure on the wetted surface the potential theory has been involved. It could be possible to reduce the problem under consideration to singular integral equations. The considered integration area specific has been taken into account. The expressions for kernels have been obtained and analysed. For testing the proposed method some analytical solutions have been applied. The convergence and effectiveness of the axially-symmetrical boundary element method have been demonstrated.

**Keywords:** elastic shell of revolution, boundary element method, fluid-structure interaction, analytical and numerical solutions

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## References

- Brebbia C. A, Telles J. C. F., Wrobel L. C. Boundary element techniques: theory and applications in engineering. *Springer-Verlag*. Berlin and New York. (1984).
- Gnitko V., Degtyariov K., Karaiev A., Strelnikova E.: Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences*. (2019). 126: 189–200. doi: 10.2495/BE420171.
- Gnitko V., Degtyarev K., Naumenko V., Strelnikova E. Coupled BEM and FEM Analysis of fluid-structure interaction in dual compartment tanks. *International Journal of Computational Methods and Experimental Measurements*. (2018). 6(6): 976–988. doi: 10.2495/CMEM-V6-N6-976-988.
- Gnitko V., Marchenko U., Naumenko V., Strelnikova E. Forced vibrations of tanks partially filled with the liquid under seismic load, *WIT Transaction on Modelling and Simulation*. (2011). vol. 52: 285–296. doi: 10.2495/BE11025.
- Gontarovskyi P., Smetankina N., Garmash N., Melezhyk I. Numerical Analysis of Stress-Strain State of Fuel Tanks of Launch Vehicles in 3D Formulation. In: Nechyporuk M., Pavlikov V., Kritskiy D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. *ICTM 2020. Lecture Notes in Networks and Systems*. (2021). vol 188: 609–619. [https://doi.org/10.1007/978-3-030-66717-7\\_52](https://doi.org/10.1007/978-3-030-66717-7_52).
- Karaiev A., Strelnikova E. Singular integrals in axisymmetric problems of elastostatics. *International Journal of Modeling, Simulation, and Scientific Computing*. (2020). 11(1): 2050003. doi:10.1142/S1793962320500038.
- Karaiev A., Strelnikova E. Axisymmetric polyharmonic spline approximation in the dual reciprocity method. *ZAMM-Journal of Applied Mathematics and Mechanics. Zeitschrift für Angewandte Mathematik und Mechanik*. (2021). vol. 101, № 1: e201800339. doi:10.1002/zamm.201800339.
- Lampart P., Gardzilewicz A., Rusanov A., Yershov S. The effect of stator blade compound lean and compound twist on flow characteristics of a turbine stage - Numerical study based on 3D NS simulations. *American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) PVP*. (1999). 397 II: 195–204.
- Misura S., Smetankina N., Misiura I. (2021). Optimal Design of the Cyclically Symmetrical Structure Under Static Load. In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. *ICTM 2020. Lecture Notes in Networks and Systems*. (2020). vol 188: 256–266. [https://doi.org/10.1007/978-3-030-66717-7\\_21](https://doi.org/10.1007/978-3-030-66717-7_21).
- Moskalenko R., Zaydenvarg O., Strelnikova O., Gnitko V. Software development for the computational analysis of crack propagation and durability of structures. *2020 IEEE KhPI Week on Advanced Technology (KhPIWeek)*. (2020): 509-514. doi: 10.1109/KhPIWeek51551.2020.9250089.
- Obaiys S. J., Ibrahim R. W., Ahmad A. F. Hypersingular Integrals in Integral Equations and Inequalities: Fundamental Review Study. In: *Differential and Integral Inequalities. Springer Optimization and Its Applications*. (2019). 151. doi.org/10.1007/978-3-030-27407-8\_25.

- Rusanov A. V., Solovey V. V, Lototskyy M. V. Thermodynamic features of metal hydride thermal sorption compressors and perspectives of their application in hydrogen liquefaction systems. *Journal of Physics: Energy*. (2020). vol. 2(2): 021007. doi: 10.1088/2515-7655/ab7bf4.
- Rusanov A. V., Kostikov A. O., Shubenko O. L., Kharlampidi D. K., Tarasova V. A., Senetskyi O. V., Highly efficient cogeneration power plant with deep regeneration based on air Brayton cycle. *J Mech Eng*. (2019). 22:1: 2–23. <https://doi.org/10.15407/pmach2019.04.012>.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova O. The Deformable and Strength Characteristics of Nanocomposites Improving. *Materials Science Forum*. Trans Tech Publications Ltd, Switzerland. (2021). Vol. 1038: 144-153. <https://doi.org/10.4028/www.scientific.net/MSF.1038.144>.
- Sierikova O., Koloskov V., Strelnikova E. The groundwater level changing processes modeling in 2d and 3d formulation. *Acta Periodica Technologica*, (2022). 53: 36-47. doi: <https://doi.org/10.2298/APT2253036S>.
- Sierikova O., Strelnikova E., Degtyariov K. Numerical Simulation of Strength and Aerodynamic Characteristics of Small Wind Turbine Blades. In: Nechyporuk, M., Pavlikov, V., Krtskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2022. *ICTM 2022. Lecture Notes in Networks and Systems*. (2023). vol 657: 357-370. [https://doi.org/10.1007/978-3-031-36201-9\\_31](https://doi.org/10.1007/978-3-031-36201-9_31).
- Sierikova O., Strelnikova E., Gnitko V., Degtyarev K. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. *IEEE 2<sup>nd</sup> KhPI Week on Advanced Technology (KhPIWeek)*. (2021): 351–356. doi: 10.1109/KhPI Week53812.2021.9570086.
- Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E. Flood risk management of Urban Territories. *Ecology, Environment and Conservation*. (2020). vol. 26 (3): 1068-1077.
- Smetankina N. V., Shupikov A. N., Sotrikhin S. Yu., Yareschenko V. G. Dynamic response of an elliptic plate to impact loading. Theory and experiment. *International Journal of Impact Engineering*. (2007). 34 (2): 264-276. <https://doi.org/10.1016/j.ijimpeng.2005.07.016>.

## **Chapter 6**

# **Construction of Fundamental Solution of {1,2}- Approximation Static Equations of Momentless Stress State for Transversely-Isotropic Plates**

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### **Abstract**

The problem of static transversely isotropic plates, which are under the action of a force was considered. Used static equations of {1,2}-approximation obtained by decomposition of the desired functions in Fourier series in Legendre polynomials with respect to the thickness of the coordinates. These equations take into account all the components of the stress tensor, including the transverse shear and normal stresses. Numerical studies demonstrating the effect of elastic constants on the components of the stress-strain state of a transversely isotropic plate.

**Keywords:** {1,2}-approximation, fundamental solution, transversely-isotropic plates, static equations, momentless stress state, integral representation, boundary element method

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## References

- Avramov K. V., Strel'nikova E. A., Pierre C. Resonant many-mode periodic and chaotic self-sustained aeroelastic vibrations of cantilever plates with geometrical nonlinearities in incompressible flow. *Nonlinear Dyn.* (2012). 70:1335–1354. <https://doi.org/10.1007/s11071-012-0537-5>.
- Bokov I. P., Strelnikova, E. A. Fundamental solution of static equations of transversely isotropic plates. *International Journal of Innovative Research in Engineering & Management.* (2015). Vol. 2, Issue-6: 56–62.
- Bondarenko N. S. The fundamental solution of differential equations of thermoelasticity {1,0}-approximation for transversely isotropic plates. *Proceedings of the Institute of Applied Mathematics and Mechanics of National Academy of Sciences of Ukraine.* (2009). V. 18: 11–18.
- Bondarenko N. S., Goltsev A. S., Shevchenko V. P. Fundamental solution {1,2}-approximation the membrane thermoelastic state transversely isotropic plates. *Reports of National Academy of Sciences of Ukraine.* (2009). № 11: 46–52.
- Bondarenko N. S., Goltsev A. S. Investigation of the influence of the environment on the state of isotropic thermoelastic plate with insulated cut at one-sided heat exchange. *Theoretical and Applied Mechanics.* (2014). № 09 (55): 42–52.
- Bondarenko N. S. Stress intensity factor in a thermoelastic bending of isotropic plates with insulated cut in the case of a symmetrical heat. Don. University. Ser. A. (2013). Pub. 2: 20–26.
- Degtyariov K., Gnitko V., Kononenko Y., Kriutchenko D., Sierikova O., Strelnikova E. Fuzzy Methods for Modelling Earthquake Induced Sloshing in Rigid Reservoirs. 2022 IEEE 3rd KhPI Week on Advanced Technology (KhPIWeek). (2022): 297-302. doi: [10.1109/KhPIWeek57572.2022.9916466](https://doi.org/10.1109/KhPIWeek57572.2022.9916466).
- Goldenveizer A. L. On the question of the calculation of shells on concentrated forces. *Appl. Mathematics and mechanics.* (1954). 8, 2: 181 – 186.
- Karaiev A., Strelnikova E. Singular integrals in axisymmetric problems of elastostatics. *International Journal of Modeling, Simulation, and Scientific Computing.* (2020). 11(1): 2050003. doi:[10.1142/S1793962320500038](https://doi.org/10.1142/S1793962320500038).
- Khizhnyak V. K., Shevchenko V. P. *Mixed problem in the theory of plates and shells: a tutorial.* Don. University. (1980). 128.
- Kiel N. A. On the action of local loads on the shell. Proceedings of the universities. *Construction and architecture.* (1973). 3: 43 – 46.
- Reissner E. Reflections on the theory of elastic plates. *Appl. Mech. Rev.* (1985). Vol. 38, № 11: 1453–1464.
- Rusanov A., Shubenko A., Senetskyi O., Babenko O., Rusanov R. Heating modes and design optimization of cogeneration steam turbines of powerful units of combined heat and power plant. *Didelių kombinuoto ciklo jėgainių kogeneracinių garo turbinų šildymo režimai ir dizaino optimizavimas. Energetika.* (2019). 65(1): 39–50.
- Sierikova O., Koloskov V., Degtyarev K., Strelnikova E. Improving the Mechanical Properties of Liquid Hydrocarbon Storage Tank Materials. *Materials Science Forum.* Trans Tech Publications Ltd, Switzerland. (2022). Vol. 1068: 223-229. doi:[10.4028/p-888232](https://doi.org/10.4028/p-888232).

- Sierikova O., Strelnikova E., Degtyarev K. Strength Characteristics of Liquid Storage Tanks with Nanocomposites as Reservoir Materials. *2022 IEEE 3<sup>rd</sup> KhPI Week on Advanced Technology (KhPIWeek)*. (2022): 151-157. doi: [10.1109/KhPIWeek57572.2022.9916369](https://doi.org/10.1109/KhPIWeek57572.2022.9916369).
- Sierikova O., Strelnikova E., Gnitko V., Degtyarev K. Boundary Calculation Models for Elastic Properties Clarification of Three-dimensional Nanocomposites Based on the Combination of Finite and Boundary Element Methods. *2021 IEEE 2<sup>nd</sup> KhPI Week on Advanced Technology (KhPIWeek)*. (2021): 351-356. doi: [10.1109/KhPIWeek53812.2021.9570086](https://doi.org/10.1109/KhPIWeek53812.2021.9570086).
- Sierikova O., Strelnikova E., Kriutchenko D., Gnitko V. Reducing Environmental Hazards of Prismatic Storage Tanks under Vibrations. *WSEAS Transactions on Circuits and Systems*. (2022). Vol. 21: 249-257. doi: [10.37394/23201.2022.21.27](https://doi.org/10.37394/23201.2022.21.27)
- Shupikov A. N., Smetankina N. V. Non-stationary vibration of multilayer plates of an uncanonical form. The elastic immersion method. *International Journal of Solids and Structures*. (2001). 38(14): 2271-2290. [https://doi.org/10.1016/S0020-7683\(00\)00166-9](https://doi.org/10.1016/S0020-7683(00)00166-9).
- Vekua I. N. A method for calculating prismatic shells. *Pr. Tbilis. Mat. Inst.* (1955). 21: 191 – 253.
- Vekua I. N. *Variational principles of the theory of shells*. Publisher of Tbilisi. (1970). 300.
- Velichko P. M., Khizhnyak, V. K., Shevchenko, V. P. Local stresses in the shells of positive, neutral and negative curvature. *X All-Union. Conf. on the theory of shells and plates*. Tbilisi: Publishing house Mitsnireba, (1975). V. 1: 31 – 41.

## **Chapter 7**

# **Boundary Element Method in Hydroelastic Interaction Problems of Structural Elements**

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### **Abstract**

Modern equipment usually operates under increased power and temperature loads. This requires determining the strength and dynamic characteristics of structural elements at the design stage in order to substantiate the reliability of operation. Experimental studies make it possible to estimate such characteristics with sufficient accuracy. However, conducting natural experiments is an expensive and not always safe procedure. Therefore, studies of the strength and vibrations characteristics of structural elements based on computer modelling are relevant. Nevertheless, the external load parameters cannot always be determined unambiguously. In this work, an effective method of analysing hydroelastic vibrations of structural elements has been developed, based on the application of potential theory methods, and elements of fuzzy logic. First, the problem of forced hydroelastic oscillations of a structural element has been solved in a deterministic formulation. It has been assumed that the fluid is ideal and incompressible, and its motion, induced by small oscillations of the elastic element, is vortex-free. Then there exists a velocity potential that satisfies the Laplace equation. The method of given modes has been

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applied, the oscillation modes of the structural element without taking into account the attached fluid masses have been chosen as the basic functions. To find the pressure of the liquid on the structural element, a hypersingular integral equation has been obtained, the solution of which has been carried out by the boundary element method, using the unknown density approximation by constant values on the boundary elements. Next, the load parameters have been fuzzified using triangular membership functions. Then the randomness of load parameters has been added to the mathematical model. Fuzzy stochastic differential equations have been obtained, which have been solved by a numerical method. The presented numerical results demonstrate the influence of the uncertainty of the initial data on the behaviour of structural elements.

**Keywords:** hydroelastic oscillations, hypersingular integral equation, boundary element method, methods of fuzzy mathematics

## References

- Andrić J., Lu D. G. Seismic hazard analysis based on fuzzy-probabilistic approach. 6th *ECCOMAS Thematic Conf. on Computational Methods in Structural Dynamics and Earthquake Engineering*. M. Papadrakakis, M. Fragiadakis (eds.), Greece, 15–17 June. (2017). doi:10.7712/120117.5739.17539.
- Avramov K. V., Strel'nikova E. A., Pierre C. Resonant many-mode periodic and chaotic self-sustained aeroelastic vibrations of cantilever plates with geometrical nonlinearities in incompressible flow. *Nonlinear Dyn.* (2012). 70:1335–1354. <https://doi.org/10.1007/s11071-012-0537-5>.
- Babenko A. E., Boronko O. O., Lavrenko Y. I., Trubachev S. I. Oscillations of non-conservative mechanical systems: monograph. National Technical University of Ukraine “Igor Sikorsky KPI.” Kyiv. (2020): 153 p.
- Chantarawichit P., Sompornjaroensuk Y., Vibration of Circular Plates with Mixed Edge Conditions. Part I: Review of Research. *ITK Research Journal*. (2020). 14(2): 136–156.
- Gnitko V., Karaiev A., Degtyariov K., Strelnikova E.: Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences*. WIT Press. (2019). 126: 189–200. doi:10.2495/BE420171.
- Gnitko V., Degtyarev K., Naumenko V., Strelnikova E. Coupled BEM and FEM Analysis of fluid-structure interaction in dual compartment tanks, *International Journal of Computational Methods and Experimental Measurements*. (2018). 6(6): 976–988. doi: 10.2495/CMEM-V6-N6-976-988.
- Gnitko V., Degtyariov K., Karaiev A., Strelnikova E. Singular boundary method in a free vibration analysis of compound liquid-filled shells. *WIT Transactions on Engineering Sciences*. (2019). vol. 126: 189–200. WIT Press: Southampton and Boston. doi: 10.2495/BE420171.

- Gontarovskyi P., Smetankina N., Garmash N., Melezhyk I. Numerical Analysis of Stress-Strain State of Fuel Tanks of Launch Vehicles in 3D Formulation. In: Nechyporuk M., Pavlikov V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2020. *ICTM 2020. Lecture Notes in Networks and Systems*. (2021). vol 188. Springer, Cham. [https://doi.org/10.1007/978-3-030-66717-7\\_52](https://doi.org/10.1007/978-3-030-66717-7_52).
- Degtyariov K., Gmitko V., Kononenko Y., Kriutchenko D., Sierikova O., Strelnikova E. Fuzzy Methods for Modelling Earthquake Induced Sloshing in Rigid Reservoirs. 2022 IEEE 3<sup>rd</sup> KhPI Week on Advanced Technology (KhPIWeek). (2022): 297-302. doi: [10.1109/KhPIWeek57572.2022.9916466](https://doi.org/10.1109/KhPIWeek57572.2022.9916466).
- Kantor B. Ya., Smetankina N. V., Shupikov A. N. Analysis of non-stationary temperature fields in laminated strips and plates. *International Journal of Solids and Structure*. (2001) 38(48/49): 8673-8684. [https://doi.org/10.1016/S0020-7683\(01\)00099-3](https://doi.org/10.1016/S0020-7683(01)00099-3).
- Karaiev A., Strelnikova E.: Singular integrals in axisymmetric problems of elastostatics / International Journal of Modeling. *Simulation, and Scientific Computing*. (2020). 11(1): 2050003. doi:10.1142/S1793962320500038.
- Rusanov A., Shubenko A., Senetskyi O., Babenko O., Rusanov R.: Heating modes and design optimization of cogeneration steam turbines of powerful units of combined heat and power plant. *Energetika*. (2019). vol. 65(1): 39-50. doi.org/10.6001/energetika. v65i1.3974.
- Salvatore F., Sarichloo Z., Calcagni D.: Marine Turbine Hydrodynamics by a Boundary Element Method with Viscous Flow Correction. *Journal of Marine Science and Engineering*. (2018). 6(2): 53. <https://doi.org/10.3390/jmse6020053>.
- Sierikova O., Koloskov V., Strelnikova E. The groundwater level changing processes modeling in 2d and 3d formulation. *Acta Periodica Technologica*. (2022). 53: 36-47. doi: <https://doi.org/10.2298/APT2253036S>.
- Sierikova E., Strelnikova E., Pisnia L., Pozdnyakova E. Flood risk management of Urban Territories. *Ecology, Environment and Conservation*. (2020). 26 (3): 1068-1077.
- Sierikova O., Strelnikova E., Degtyarev K. Seismic Loads Influence Treatment on the Liquid Hydrocarbon Storage Tanks Made of Nanocomposite Materials. *WSEAS Transactions on Applied and Theoretical Mechanics*. (2022). vol. 17: 62-70. doi: [10.37394/232011.2022.17.9](https://doi.org/10.37394/232011.2022.17.9).
- Sierikova, O., Strelnikova, E., Degtyariov, K.: Numerical Simulation of Strength and Aerodynamic Characteristics of Small Wind Turbine Blades. In: Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2022. *ICTM 2022. Lecture Notes in Networks and Systems*. (2023). 657. Springer, Cham. [https://doi.org/10.1007/978-3-031-36201-9\\_31](https://doi.org/10.1007/978-3-031-36201-9_31).
- Sierikova E., Strelnikova E. and Yakovlev V. Mathematical model of dangerous changing the groundwater level in Ukrainian industrial cities. *Journal of Environment Protection and Sustainable Development*. (2015). vol. 1: 86-90. /Files/journals/JTE/V3No1/StrelnikovaE.pdf.
- Wang Z. W., Ge N., Li C. W. Structural Vibration Mode Fuzzy Control Based on BPNeural Network Algorithm. *Journal of Shandong University (Engineering Science)*. (2020). vol. 50, 243(05): 17-23.
- Zadeh L. A. Fuzzy sets, *Information and Control*. (1965). vol. 8: 338-353.