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DETERMINATION OF THE LARGEST LYAPUNOV EXPONENT OF CHAOS IN THE DYNAMICS OF HAZARDOUS PARAMETERS OF A GAS ENVIRONMENT FOR THE RAPID IGNITION DETECTION

The object of research is the largest Lyapunov exponent of the dynamics of hazardous gas environment parameters in premises at intervals of reliable absence and presence of ignition of materials in premises. The problem is to determine and develop a strategy for using the largest Lyapunov exponent on a one-dimensional sample of real contaminated measurements of hazardous gas environment parameters in premises for the prompt detection of material ignitions. An experimental verification of the determination of the largest Lyapunov exponent of the dynamics of the main hazardous gas environment parameters during ignition of materials in a laboratory chamber at intervals of reliable absence and occurrence of ignition was performed. It was established that during ignition of materials, the values of the largest Lyapunov exponent indicate a decrease in stability and a transition to chaos in the dynamics of temperature and carbon monoxide concentration for all the test materials under study. This indicates a loss of the degree of "order" in the dynamics of temperature and carbon monoxide concentration. At the same time, the value of the largest Lyapunov exponent of the dynamics of the specific optical density of smoke does not change significantly and remains stable with some decrease in stability during ignition of the material. It was found that the use of such a parameter for detecting the ignition of materials has significant advantages in the case of using the dynamics of temperature and carbon monoxide concentration of the gas environment of the premises. The results obtained are useful from a theoretical point of view for determining the largest Lyapunov exponent for a one-dimensional sample of real contaminated measurements for an arbitrary dangerous parameter of the gas environment at an arbitrary observation interval. The practical significance lies in the possibility of further improving existing fire protection systems of objects in order to prevent fires.

Keywords: *largest Lyapunov exponent, operational detection of ignition, dangerous parameters of the gas environment, premises.*

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1. Introduction

Safety is a fundamental need of humanity. Fires are one of the causes of human safety violations. Such events lead to the death and injuries of people, damage or completely destroy property and various objects [1]. An effective way to fight fires is to detect them at an early stage of development, that is, at the stage of ignition of materials (IM). In the case of uncontrolled development of a fire, its extinguishing becomes difficult, economic losses increase [2], and colossal environmental damage is also caused [3]. Therefore, the operational IM detection is one of the main components of ensuring the fire safety of various objects. In case of IM and fire in the premises, the hazardous parameters (HP) are, first of all, the temperature, concentration of carbon monoxide and the specific optical density of the smoke of the gas environment (GE) [4]. In this regard, the operational ignition detection based on the characteristics of the dynamics indicated by GE HP is an important direction in ensuring the fire safety of premises. The peculiarities of the GE HP dynamics of fire protection systems used in the existing fire protection systems do not allow for quick IM detection in the premises [5]. Therefore, it is relevant to search for peculiarities of the dynamics of the gas environment in the premises, which allow IM detection. To such features of the GE HP dynamics, one can attribute its chaotic indicators. The chaotic GE HP dynamics can be caused by the peculiarities of physical processes that are difficult for mathematical description, taking place in the gaseous environment under the influence of irreversible physicochemical processes in the burning materials themselves, as well as in the enclosing structures of the premises.

In this regard, the search for methods of identifying the specified features of the GE HP dynamics will allow in practice to detect the ignition of materials in the premises of objects and prevent the occurrence of fires with losses.

To study the chaotic dynamics of various experimental data, a well-known tool based on Lyapunov exponents has been developed [6, 7]. In real conditions, the GE HP dynamics at IM is non-linear with varying degrees of chaos. In these conditions, the Lyapunov indicators at different time intervals of the GE HP dynamics will be different. In this

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regard, the study of the largest Lyapunov exponent on the dynamics intervals corresponding to the reliable absence and presence of IM is relevant from the point of view of the operational IM detection in the premises. In [8], an approach to the operational IM detection is considered based on one arbitrary GE HP, measured by a set of sensors in the premise, followed by network data processing. The limitation of this approach is the use of traditional features of dynamics without taking into account its nonlinearity and chaos. The traditional peculiarities of the GE HP dynamics in the IM interval due to low energy are usually analogous to the interval of no ignition. Differences arise only at the stage of a developed fire. A similar approach in the case of simultaneous measurement of different GE HPs is considered in [9]. At the same time, consideration is limited to traditional features of dynamics with their inherent limitations. In these works, the peculiarities of the dynamics inherent in the Lyapunov exponents are not considered or explored. In [10], the application of the spectral features of the dynamics of the main GE HPs for the operational IM detection in a laboratory chamber is investigated. Spectral features of dynamics based on the direct discrete Fourier transform for data on intervals of reliable absence and presence of IM allows to determine instantaneous amplitude and phase spectra of dynamics for the considered time intervals. However, the application of a discrete Fourier transform significantly narrows the class of permissible GE HP dynamics, which reduce the possibility of operational detection of real ignitions. The peculiarities of the real GE HP dynamics in the form of the rate of heat release and the intensity of plantation wood burning are studied in [11].

The study of the rate of heat release from the intensity of wood burning was carried out in [12]. However, in this work, the results are limited to the study of the average values of the rate of heat release and the intensity of combustion. Analogous studies for organic glass and cypress are carried out in [13]. However, in works [11–13], the Lyapunov indicators of the GE HP dynamics, which are capable of revealing its chaotic nature in the case of IM in the premises, are not considered or studied. The application of third-order spectra of the GE HP dynamics for the operational IM detection is considered in [14]. It is shown that such spectra make it possible to reveal the degree of correlation between frequency components caused by the nonlinearity of the GE HP dynamics. However, the degree of correlation of the frequency components significantly depends on the HP energy. However, the use of third-order spectra is quite difficult to implement and has significant limitations, since it is based on the Fourier transform. Peculiarities of using the average bicoherence of the GE HP dynamics for operational IM detection are discussed in [15]. It is noted that the average value of bicoherence can be used for the operational IM detection. A disadvantage of the use of average bicoherence should be considered its alternating character, which makes it difficult to interpret the degree of chaos of the studied GE HP dynamics at IM. In [16], the peculiarities of determining bicoherence for one implementation and an ensemble of data implementations are studied. It is noted that the definition of bicoherence by the ensemble of realizations is the most effective from the point of view of the operational IM detection. However, the definition of bicoherence turns out to be quite difficult to implement and has certain limitations due to the Fourier transform. At the same time, Lyapunov indices are not considered or studied. The use of the empirical cumulative distribution function of the current recurrence of the GE HP state in the premises for the operational IM detection is considered in [17].

However, the application of the empirical cumulative distribution function is connected with the implementation of a complex computational procedure. In [18], the application of the probability of the absence of recurrence of the vector-incremented GE HP based on the empirical cumulative distribution function is explored. However, the implementation of this method is also associated with complex calculations, which significantly limits its use in practice. At the same time, the non-linear connection of the frequency components in the bispectrum of the vector-incremented GE HP is not taken into account. The results of the study of the GE HP dynamics during IM fire tests are considered in [19]. It is noted that for the reliable IM detection, it is necessary to take into account the joint dynamics of the CO concentration and the specific optical density of smoke. At the same time, Lyapunov indices are not considered as a sign of IM detection. In [20], the results of an experimental study of the mutual connection between various GE HPs at IM are presented. However, mutual connection is estimated by correlation, which takes into account only the degree of linear connection. Lyapunov exponents, which make it possible to reveal nonlinear features of the dynamics of various GE HPs [21], are not considered.

The object of research was the largest Lyapunov exponent for characteristic intervals of the GE HP dynamics during ignition of materials.

Thus, it is interesting to use the bispectrum of the GE HP dynamics for operational IM detection. However, the application of the bispectrum of the GE HP dynamics for the operational IM detection is currently insufficiently researched. In connection with this, the strategy of using the largest Lyapunov exponent to estimate the chaotic GE HP dynamics on the intervals of reliable presence and absence of IM is of special interest. Therefore, the study of the largest Lyapunov indicator of the GE HP dynamics in the absence and presence of lit objects should be considered an important and unsolved part of the problem of operational IM detection in premises.

The aim of research is to develop a strategy for determining the largest Lyapunov exponent for the dynamics of the main hazardous parameters of the gas environment when materials ignite in the premises.

Research tasks:

1. To develop a method for calculating the Lyapunov exponent based on a discrete finite sample of the observations of one of the GE HPs during the ignition of materials.
2. To experimentally verify the method using the example of the observed GE HP during TM ignition in a laboratory chamber.

2. Materials and Methods

Fires in premises occur with the participation of materials that undergo not only an ignition reaction, but a smoldering reaction that often develops very slowly. Compared to flaming fires, smoldering fires release a relatively small amount of thermal energy [22], but, on the other hand, after the start of the burning of materials, it leads to a significant increase in the concentration of gases and volatile organic compounds [23, 24]. In addition, combustion products may contain irritating gases that reduce the likelihood of rescue. Released toxic volatile substances and suffocating gases can pose a danger to people before the appearance of thick smoke and open flames. Accumulation of carbon monoxide can probably disable it faster than smoke completely filling the premise [25]. In fact, inhalation of toxic combustion products is the main cause of death in accidental fires, even more often than burns [26, 27]. Over the past 30 years, the use of synthetic materials in building materials, furniture and insulation of electrical wires has increased significantly. These materials produce more toxic substances, especially in the presence of flame retardants [28]. Correct detection of toxic compounds can lead to faster IM detection and increase the level of safety of people and premises.

It was assumed that any burning of the material causes a change in its physical properties, which affects the dynamics of the main GE HP in the premise, such as temperature, concentration of toxic combustion products, and also smoke density. These changes are directly related to heat release, the mass rate of material burning, and the rate of flame propagation. Following [22–29], different amounts of toxic combustion products and heat are released at different times at the IM in the premise GE. This means that the operational IM detection can be carried out on the basis of the peculiarities of the temporal dynamics of the corresponding GE HP. In practice, the ignition of materials is characterized by the complexity and uncertainty of physicochemical reactions and mechanisms of interaction of ignition products with the parameters of the indoor environment. In addition, at the beginning of a fire, the level of the specified reactions and mechanisms is usually not sufficient to use them for prompt ignition detection. In this case, it is proposed to consider the GE of premises as a complex dynamic system, which in general can be deterministic, chaotic or random. At the same time, IM will lead to a change in the properties of such a system. Therefore, it is possible to perform the operational IM detection on the basis of changes in the properties of the system compared to its properties before ignition. This problem can be solved on the basis of Lyapunov indicators of the GE HP dynamics at characteristic time intervals. At the same time, Lyapunov indices can be considered as an effective tool for identifying the properties of unknown complex dynamic systems of various physical nature [6].

It is known that chaos is a fundamental nonlinear property for most real dynamical systems [30, 31]. Nonperiodic chaotic oscillations are the most typical for various real dynamic systems [32, 33]. In general, the largest Lyapunov exponent, which characterizes the property of dynamic systems, is determined over an infinite time interval. However, in practice, the time interval of observation of real dynamic systems is usually finite. In addition to this observation, they are subject to noise. All of this ultimately complicates the accuracy of determining the Lyapunov exponent and its interpretation [34]. The Lyapunov exponent characterizes the speed of the run-up of infinitely close trajectories. Quantitatively, the two trajectories in the phase space with the initial divergence vector diverge (provided that the divergence can be considered within the framework of the linearized approximation) with an exponential rate. At the same time, the maximum Lyapunov exponent characterizes the predictability of a dynamic system, and its positive value is usually considered a sign of system chaos (if certain other conditions are met). The research materials were the results of monitoring the output signals of the sensors measuring the main GE HPs. For this, a laboratory chamber simulating a leaky premise was created. The dimensions of the camera were: 1500 × 1000 × 500 mm. Sensors measuring temperature, specific optical density of smoke, and CO concentration in the GE of the chamber were placed in the ceiling area of the chamber in the absence and presence of ignition of test materials (TM) [35]. GE HP measurements at intervals of reliable absence and presence of TM ignition were made at discrete moments of time with an interval of 0.1 second. Alcohol, paper and textiles were chosen as TM. The choice of specified TMs was determined by their different specific mass rate of burning [36]. The measured values of the output signals of the corresponding sensors were characterized by the corresponding sets $\{x(k)\}$ of data, which were stored in the computer's memory for their subsequent processing in order to determine the Lyapunov exponent for each of the characteristic intervals. Each set of measurement data $\{x(k)\}$ was determined by discrete measurements $x_1, x_2, x_3, ..., x_N$ of the corresponding GE HP at the observation interval. Here, N defines the moment of end of GE HP observation. The value N for intervals of reliable absence and presence of IM was chosen equal to two hundred counts. The gas temperature in the chamber was measured with a DS18B20 sensor (USA), the specific optical density of smoke was measured with an IPD-3.2 sensor (Ukraine), and the CO concentration was measured with a Discovery sensor (Switzerland) [37].

The applied method was based on the calculation of a chaoticity measure of the dynamic system under study, which generates a change in the GE HP during the IM, for discrete measurements $x_1, x_2, x_3, ..., x_N$ of the observed GE HP.

3. Results and Discussion

As a measure of the chaotic nature of discrete measurements of the observed GE HP, it is proposed to use the Lyapunov exponent, which determines the rate of divergence of trajectories in phase space. A positive value of the Lyapunov exponent will indicate chaotic behavior, while zero or negative values indicate system stability. For discrete measurements $x_1, x_2, x_3, ..., x_N$, the largest Lyapunov exponent is usually calculated, which characterizes the presence or absence of chaos in the system. In general, the Lyapunov exponent is a mathematical characteristic of dynamic systems that describes the rate of divergence or convergence of close trajectories in the corresponding phase space. To calculate the Lyapunov exponent from a discrete sample of one of the system state parameters, special algorithms are used that estimate the average rate of exponential divergence of close trajectories in phase space. The results of an analysis of known algorithms and the features of their calculation of the Lyapunov exponent are contained in [38, 39]. The key to the known algorithms is the reconstruction of the phase space of the dynamic system under study from a single discrete sample using the delay method, the selection of a suitable algorithm [40–44], and the estimation of the largest Lyapunov exponent, a positive value of which indicates the chaos of the dynamic system generating the sample. The important aspects of using the known algorithms are: the data volume (the accuracy of determining the Lyapunov exponent depends on the sample size), the embedding parameters (the embedding dimension and the delay time significantly affect the accuracy of the Lyapunov exponent). Among the known algorithms [40–44], only two allow estimating the largest Lyapunov exponent from a data sample without knowing the operator generating this sample. These are the Kantz [42, 38, 45] and Rosenstein [41] algorithms. The Kantz algorithm is based on calculating the average distance between neighboring points over time and estimating the largest Lyapunov exponent by fitting the function of changing the current average distance between neighboring points to an exponential function. In this case, the Kantz algorithm allows one to directly estimate the largest Lyapunov exponent. The Rosenstein algorithm is also based on calculating the average distance between adjacent points over time, but the largest Lyapunov exponent is estimated by the slope of the function of change in the current average distance between adjacent points on a double logarithmic scale. The advantage of the Rosenstein algorithm is that it calculates the average distance between several adjacent points. This ensures increased robustness of the algorithm to noisy data. The Kantz-Rosenstein algorithm [43], which is a modification of the indicated algorithms and combines the advantages of the above algorithms, is also known. However, the use of the algorithms [41–43] presupposes a priori knowledge of the embedding parameters or their determination from a data sample. The solution to the latter problem presents difficulties, which increase significantly in the case of noisy data. In addition, it is necessary to know in advance the state of the dynamic system under study and select the starting point at the beginning of the region of the state under study. In the case under consideration, the specified conditions are not met, since the one-dimensional data sample $x_1, x_2, x_3, ..., x_N$ is noisy and is taken from the dynamic domain of a system with an unknown state. In this regard, under the conditions under consideration, it is proposed to estimate the largest Lyapunov exponent for an arbitrary start T_0 and the size of the one-dimensional data sample $x_{1+To}, x_{2+To}, x_{3+To}, ..., x_{N+To}$, where $N + T_o = T_k$ – end of the interval under study, by the function $S(\tau, m, T_o, T_k)$ of the following form

$$S(\tau,m,To,Tk)=\frac{1}{Tk-To-m}\sum_{t=To}^{Tk-m}\ln\left(\frac{1}{m}\sum_{i=1}^{m}\left|x_{t+\tau}-x_{i+\tau+To}\right|+a\right), \quad (1)$$

where m, τ – the embedding dimension and delay; a – a known constant less than unity, necessary for calculating the natural logarithm in the case $\frac{1}{m}\sum_{i=1}^{m}\left|x_{t+\tau}-x_{i+\tau+To}\right|=0$. In the case of measuring a single arbitrary GE HP ($m = 1$ and $\tau = 0$), expression (1) takes the form

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$$S(0,1,To,Tk)=\frac{1}{Tk-To-1}\sum_{t=To}^{Tk-1}\ln\left(\left|x_t-x_{To+1}\right|+a\right). \quad (2)$$

Representation (2) allows one to investigate the estimate of the largest Lyapunov exponent based on a fixed sample of noisy measurements of a single parameter of an unknown dynamic system with an arbitrary start and end point, without using the embedding operation. This is explained by the fact that the state of the observed unknown dynamic system at an arbitrary point in time is reflected as a corresponding point characterized by the value of a single measured parameter of the system. While a single parameter does not fully characterize the property of the entire system, it nevertheless allows one to unambiguously study the property of the system based on this parameter. Moreover, the properties of the system based on different parameters may differ. Therefore, the gaseous medium as a single dynamic system will have different properties with respect to its various parameters. In this regard, (2) allows to investigate the randomness of an arbitrary measured parameter of a gas environment upon the IM occurrence. This assumption was verified experimentally by calculating the value of (2) based on a sample of measurements of the main GE HP during intervals of reliable absence and reliable presence of ignition of various TMs.

The results of studying the largest Lyapunov exponent (2) for the considered main GE HP in a laboratory chamber during the interval of reliable absence of TM ignition are presented in Table 1. For comparison, Table 2 presents the results of studying a similar exponent during the interval of reliable TM ignition.

Table 1

Values of the largest Lyapunov exponent (2) for CO, smoke, and temperature in the interval of reliable absence of ignition of materials

GE HP/TM	ALCOHOL	PAPER	WOOD	TEXTILES
CO	–0.224	–1.606	–1.949	–1.534
Smoke	–2.239	–1.918	–1.949	–2.061
Temperature	–2.172	–0.908	–1.642	–1.528

Table 2

Values of the largest Lyapunov exponent (2) for CO, smoke, and temperature in the interval of reliable presence of material ignition

GE HP/TM	ALCOHOL	PAPER	WOOD	TEXTILES
CO	+1.489	+0.134	+0.187	–1.132
Smoke	–1.711	–1.551	–2.024	–1.254
Temperature	+1.395	+1.126	–0.238	–0.009

From the analysis of the results (Table 1), it follows that in terms of the parameters of CO concentration, specific optical density of smoke, and GE temperature in the chamber is stable as a dynamic system (all values of the largest Lyapunov exponent are negative). At the same time, for different GE HP, the values of the largest Lyapunov exponent are not the same. This means that the GE in the chamber as a dynamic system in the initial state is more stable in terms of the specific optical density of smoke visibility and temperature (–2.2) than in terms of CO concentration (–0.2). From Table 2 it is evident that TM ignition leads to a change in the properties of the dynamics of the studied HPs. Moreover, the changes in these properties are not the same. The dynamics of CO concentration and GE temperature during alcohol ignition in the chamber become chaotic. At the same time, the property of the dynamics of the specific optical density of smoke changes insignificantly and is characterized by stability on the whole, but the margin of this stability is reduced. The ignition of paper and wood causes chaotic dynamics of the CO concentration. However, textile combustion within the studied interval has virtually no effect on the pattern of CO concentration dynamics. The specific optical density of smoke dynamics remains virtually unchanged during combustion of all TMs and is characterized by stability (negative Lyapunov exponent). However, temperature dynamics become chaotic during alcohol and paper combustion. However, during wood and textile combustion, temperature dynamics within the analysis interval remain stable, although the stability margin for the Lyapunov exponent generally decreases. Thus, the obtained study results convincingly demonstrate that the Lyapunov exponent can generally be considered as an indicator of the rapid IM detection.

Moreover, the highest sensitivity of the proposed Lyapunov exponent is observed for such GE HP in the chamber as CO concentration and temperature. It should be noted that the obtained results and conclusions apply to fixed-duration (20-second) GE HP measurement intervals. Therefore, it can be assumed that with increasing duration of these intervals, the Lyapunov exponent will reflect the chaotic dynamics of the measured GE HP during IM, which are characterized by a low ignition rate and pyrolysis.

Limitation of this research is the use of only CO concentration, specific optical density of smoke, and GE temperature in a laboratory chamber for a fixed set of ignition materials during experimental verification of the method at specified intervals of reliable absence and presence of ignitions. Therefore, *further research* should be aimed at investigating the Lyapunov exponent for an expanded set of GE HP during fire tests in real spaces and under varying fire loads.

4. Conclusions

1. A measure for calculating the largest Lyapunov exponent is proposed. It is based on a finite discrete sample of observations of one arbitrary hazardous parameter of the gas environment during ignitions of various materials. This measure can be used to identify the chaotic nature of the dynamics of the main GE HPs during material ignition. It is shown that in the absence of material ignition, the value of this measure indicates a stable nature of the GE HP dynamics. When materials ignite, the stable nature of the dynamics changes to a chaotic one. It is proposed to use the proposed measure as a possible indicator for the rapid detection of material ignitions in premises.

2. An experimental test of the proposed indicator for detecting TM ignitions was performed based on the value of the largest Lyapunov exponent of the dynamics for CO concentration, specific optical density of smoke, and GE temperature in a laboratory chamber, observed during intervals of reliable absence and presence of ignition. It was found that material ignitions lead to a decrease in stability and a transition to chaotic dynamics of the CO concentration and GE temperature. At the same time, the measure of the dynamics of the specific optical density of smoke for the studied IM indicates an insignificant change, which generally characterizes its stability during ignition.

Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship or other, that could influence the research and its results presented in this article.

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Data availability

Data will be made available on reasonable request.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies in creating the submitted work.

Authors contributions

Igor Tolok: Project administration; ***Boris Pospelov***: Software; ***Evgeniy Rybka***: Methodology; ***Serhii Savchenko***: Formal analysis; ***Yurii Kozar***: Investigation; ***Olekcii Krainiukov***: Supervision; ***Konstantin Sporyshev***: Data curation; ***Larysa Maladyka***: Writing – original draft; ***Vyacheslav Surianinov***: Resources; ***Maksym Harifullin***: Conceptualization.

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