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**EDİTÖRLER:**  
Prof. Dr. Aytekin DEMİRCİOĞLU  
Prof. Dr. Beyhan ZABUN

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## **BOUNDARY ELEMENT METHOD IN INVESTIGATION OF FLUID OSCILLATIONS IN SHELLS OF REVOLUTION WITH AND WITHOUT BAFFLES**

**PhD K.G. Degtyarev**

Institute of Power Machines and Systems, National Academy of Sciences of Ukraine,  
V. N. Karazin Kharkiv National University, Ukraine,

**PhD Denys Kriutchenko**

Institute of Power Machines and Systems, National Academy of Sciences of Ukraine,  
**PhD student M.S. Mishchenko**

V. N. Karazin Kharkiv National University

**DSc Olena Sierikova**

Hon, Prof, M.S. Bokarius Scientific Institute, Ukraine

**DSc, Professor Elena Strelnikova\***

Institute of Power Machines and Systems, National Academy of Sciences of Ukraine  
V. N. Karazin Kharkiv National University,

Kharkiv National University of Radio Electronic.

### **Abstract**

This work focuses on the mathematical modeling and numerical analysis of fluid oscillations in axisymmetric shells of revolution, specifically conical and spherical geometries, both with and without internal horizontal baffles. The primary objective of the study is to develop and validate a boundary element method to solve three-dimensional boundary value problems governing the motion of an ideal incompressible fluid.

The proposed boundary element framework allows for an efficient and accurate assessment of influencing the internal baffles and their geometric parameters on the natural frequencies and mode shapes of fluid oscillations. This capability is particularly important for understanding sloshing behavior and mitigating its adverse effects in engineering systems. The methodology provides both theoretical advancement and practical significance in the area of fluid–structure interaction modeling.

Future investigations will aim on extending the present model to incorporate strong seismic loading, nonlinear dynamic effects, and more sophisticated fluid-structure coupling mechanisms, thereby improving the accuracy and reliability of predictions for complex real-world engineering systems.

### **1. Introduction**

Shell structures with compartments partially filled with liquid are widely used in modern industry, particularly in the aerospace, oil and gas, energy, and transportation areas. These include rocket fuel tanks, marine and spacecraft tanks, aircraft tanks, as well as ground-based storage tanks for water, oil, liquefied gas, and other substances. Experimental investigations of such dampers are costly and may be accompanied by specimen failure and undesirable environmental consequences. Therefore, computer modelling of fluid dynamics in tanks and reservoirs becomes an effective tool for virtual testing, enabling the optimization of tank designs with dampers and ensuring their reliability under operating conditions (Choudhary N. et al., 2025).

In the early stages of stability analysis of tanks partially filled with liquid, methods based on relatively simple models and approaches were used (Housner, 1957). Despite their simplicity,

these methods made it possible to understand the complex dynamics of fluid motion in tanks and fuel containers and to identify certain possibilities for optimizing structural parameters (Veletsos A. S., 1976).

In recent years, modern computational methods have been widely used for analyzing the strength and vibrations of structures interacting with fluids, in particular the Finite Element Method (FEM) (Shvets A. et al., 2025), and (Murawski K. et al., (2021), the Boundary Element Method (BEM) (Gnitko V.I. et al., 2022), meshless methods (Smetankina N. et al, 2021), finite difference method (Murawski K., 2020), hypersingular integral equation method (Strelnikova E. et al., 2024) and series expansion methods (Smetankina N. et al., 2023).

A number of effective means for reducing the amplitude of liquid sloshing have been proposed, including the use of horizontal (Strelnikova E. et al., 2020) and vertical baffles (Strelnikova E. et al., 2019), floating roofs (Choudhary N. et al., 2021), and advanced materials for manufacture (Sierikova O. et al., 2022<sup>a,b,c</sup>), the effectiveness of which has been confirmed by both numerical and experimental studies. Experimental studies (Martinez-Carrascal J. et al., 2021) revealed the influence of hydrodynamic damping under vertical excitation using single-degree-of-freedom tank models, which made it possible to qualitatively analyse the stabilizing effect of damping on the dynamic characteristics of the system.

In this study, numerical methods for solving spectral boundary value problems have been improved to determine the fundamental frequencies and mode shapes of liquid oscillations in a rigid shell of revolution with changing free surface. The obtained modes were used as basis functions to study forced liquid oscillations in the tank under coupled action of horizontal and vertical excitations.

## 2. Problem statement

The aim of the study is to develop a numerical method for studying the stability of fluid motion in tanks, taking into account the presence of a horizontal baffle. A rigid spherical shell with a horizontal baffle is considered (Fig. 1). The influence of the baffle on the natural frequencies and mode shapes of the coupled “shell-fluid” system is investigated, as well as its effect on the free surface elevation under coupled periodic excitations in the horizontal and vertical directions.

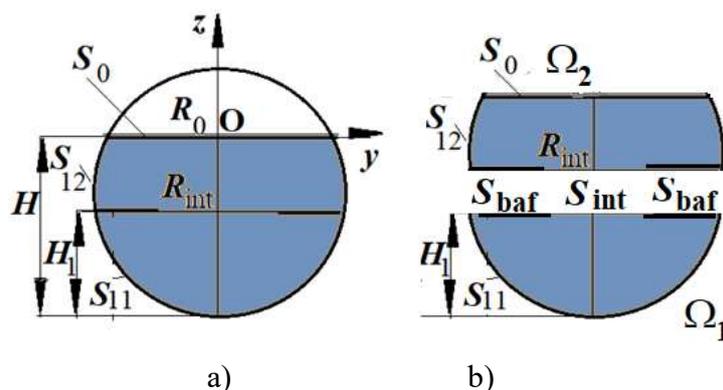


Figure 1. Spherical shell and sub-domains

Let  $S_1$  denote the wetted surface of the shell, whereas  $S_0$  is the liquid free surface. It should be noted that the surface  $S_1$  includes the wall parts  $S_{11}$  and  $S_{12}$ , as well as the baffle surface  $S_{baf}$ , which is installed inside the shell at the height  $H_1$ .

The liquid is assumed to be inviscid and incompressible, and its motion is irrotational; capillary effects are neglected. Since the flow is irrotational, there exists a scalar velocity potential  $\Phi$  such that  $\mathbf{V} = \nabla\Phi$ , where  $\mathbf{V} = (V_1, V_2, V_3)$  is the velocity vector. In the domain  $\Omega = \{0 \leq r \leq R, -H \leq z \leq 0\}$ , the potential  $\Phi$  satisfies the Laplace equation  $\nabla^2\Phi(x, t) = 0$ .

The following boundary conditions are imposed for the Laplace equation:

$$\left. \frac{\partial\Phi}{\partial\mathbf{n}} \right|_{S_1} = 0, \quad \left. \frac{\partial\Phi}{\partial\mathbf{n}} \right|_{S_0} = \frac{\partial\zeta}{\partial t}, \quad \frac{p-p_0}{\rho_l} = -\frac{\partial\Phi}{\partial t} - (g + a_v(t))\zeta + a_h(t)x \Big|_{S_0} = 0. \quad (1)$$

Here,  $\mathbf{n}$  is the unit outward normal to the surface,  $p_0$  is the atmospheric pressure,  $p$  is the liquid pressure, and  $a_v(t)$ ,  $a_h(t)$  are the accelerations of the forcing in the vertical and horizontal directions, respectively,  $\zeta = \zeta(x, y, t)$  is an unknown function describing the time-dependent variation of the liquid free surface.

Thus, the study of liquid sloshing in a rigid shell reduces to determining two unknown functions,  $\Phi$  and  $\zeta$ , by solving the following boundary value problem:

$$\nabla^2\Phi = 0, \quad \left. \frac{\partial\Phi}{\partial\mathbf{n}} \right|_{S_1} = 0, \quad \left. \frac{\partial\Phi}{\partial\mathbf{n}} \right|_{S_0} = \frac{\partial\zeta}{\partial t}, \quad p - p_0 \Big|_{S_0} = 0. \quad (2)$$

To solve problem (2), a combination of the mode superposition approach and integral equation method within the framework of the BEM is employed.

As in (Strelnikova E. et al., 2020), the unknown functions are represented as follows:

$$\begin{aligned} \Phi(x, y, z, t) &= \sum_{k=1}^n \dot{d}_k(t) \varphi_k(x, y, z), \\ \zeta(x, y, t) &= \sum_{k=1}^n d_k(t) \zeta_k(x, y), \end{aligned} \quad (3)$$

where the basic functions  $\varphi_k(x, y, z)$ ,  $\zeta_k(x, y)$  are solution of the following spectral boundary value problem (Degtyariov K. et al., 2024):

$$\nabla^2\varphi_k = 0, \quad \left. \frac{\partial\varphi_k}{\partial\mathbf{n}} \right|_{S_1} = 0, \quad \left. \frac{\partial\varphi_k}{\partial\mathbf{n}} \right|_{S_0} = \frac{\chi_k^2}{g} \varphi_k \Big|_{S_0}. \quad (4)$$

Here,  $\chi_k$  are the natural frequencies corresponding to the mode shapes  $\varphi_k$ . Assuming that the free surface position at the initial moment corresponds to  $z = 0$ , the free surface elevation  $\zeta(x, y, t)$  can be expressed as:

$$\zeta(x, y, t) = \frac{1}{g} \sum_{k=1}^n \chi_k^2 d_k(t) \varphi_k(x, y, 0), \quad (5)$$

where  $g$  is the acceleration due to gravity, and  $d_k(t)$  are the generalized coordinates. These basis functions are obtained by solving the system of singular integral equations as it was proposed in (Brebbia C. A. et al., 1992).

### 3. The Mathieu system of differential equations

For studying the fluid motion in shells of revolution, the following expression for pressure is used (Gani E. et al., 2025):

$$\frac{p}{\rho_l} = -\frac{\partial\Phi}{\partial t} - (g + a_v(t))z + a_h(t)x + \frac{p_0}{\rho_l}, \quad (6)$$

where  $g$ ,  $a_h(t)$ , and  $a_v(t)$  are the accelerations due to gravity and in the horizontal and vertical directions, respectively,  $\rho_l$  is the fluid density, and  $p_0$  is the atmospheric pressure. It

leads to the following system of the ordinary equations - the Matheu system, (Kovacic I. et al., 2018)

$$\begin{aligned} \ddot{d}_{k0}(t) + \omega_{k0}^2 \left(1 + \frac{a_v(t)}{g}\right) d_{k0}(t) &= 0, \\ \ddot{d}_{k1}(t) + \omega_{k1}^2 \left(1 + \frac{a_v(t)}{g}\right) d_{k1}(t) + a_h(t)F_{k1} &= 0, F_{k1} = \frac{(r, \varphi_{k1})}{(\varphi_{k1}, \varphi_{k1})}, \\ \ddot{d}_{kl}(t) + \omega_{kl}^2 \left(1 + \frac{a_v(t)}{g}\right) d_{kl}(t) &= 0, k = \overline{1, n}; l = \overline{2, m} \end{aligned} \quad (7)$$

with the initial conditions

$$d_{kl}(0) = d_{kl}^0, \dot{d}_{kl}(0) = \dot{d}_{kl}^1, k = \overline{1, n}, l = \overline{0, m}. \quad (8)$$

The Matheu system (7) together with initial conditions (8) is applied for numerical analysis of the liquid sloshing in the spherical shells with and without baffles.

#### 4. Numerical results and discussion

As a benchmark test, a spherical shell without baffles of radius  $R = 1$  m, partially filled with an ideal incompressible fluid to a depth  $H_1$  (Fig. 1), is considered. To solve the spectral boundary value problem (4), boundary elements with constant density approximation were used (Brebbia C. A. et al., 1992). Different numbers of boundary elements were chosen along the meridian ( $N_G$ ), and along the radius of the free surface ( $N_0$ ). Table 1 presents the results for the natural frequencies of axisymmetric liquid oscillations in the specified tank at different filling levels  $H_1$ .

Table 1. Natural frequencies of axisymmetric ( $l = 0$ ) oscillations of the liquid free surface in a spherical tank

| k | Method                       | Filling level $H_1, m$ |       |        |        |       |        |
|---|------------------------------|------------------------|-------|--------|--------|-------|--------|
|   |                              | 0.2                    | 0.6   | 1.0    | 1.4    | 1.8   | 1.99   |
| 1 | (Mciver P.,1989)             | 3.826                  | 3.650 | 3.740  | 4.245  | 6.764 | 29.050 |
|   | BEM, $N_\Gamma=50, N_0=25$   | 3.830                  | 3.649 | 3.749  | 4.240  | 6.763 | 29.028 |
|   | BEM, $N_\Gamma=100, N_0=50$  | 3.826                  | 3.651 | 3.744  | 4.244  | 6.764 | 29.068 |
|   | BEM, $N_\Gamma=260, N_0=130$ | 3.826                  | 3.650 | 3.745  | 4.245  | 6.764 | 29.071 |
| 2 | (Mciver P.,1989)             | 9.256                  | 7.265 | 6.976  | 10.012 | 12.11 | 51.812 |
|   | BEM, $N_\Gamma=60, N_0=30$   | 9.267                  | 7.268 | 6.977  | 10.015 | 12.12 | 52.025 |
|   | BEM, $N_\Gamma=130, N_0=65$  | 9.257                  | 7.266 | 6.976  | 10.014 | 12.12 | 52.000 |
|   | BEM, $N_\Gamma=260, N_0=130$ | 9.266                  | 7.265 | 6.9767 | 10.013 | 12.11 | 51.815 |

As shown by the data in Table 1, the proposed BEM demonstrates convergence with an increasing number of boundary elements. Further increasing the number of boundary elements did not lead to significant changes in the results. The data obtained by this method are in good agreement with the results reported in (Mciver P.,1989).

Next, a spherical shell of radius 1 m with a filling level of  $H = 1.4$  m is considered, with a baffle located at  $H_1 = 1.0$  m. Internal horizontal baffles are examined, characterized by the interface surface radius  $R_{int}$ ; the baffle radius is  $R_b = R - R_{int}$ . The first three natural frequencies for mode  $l = 1$  were computed for  $R_{int} = 0.2$  m,  $R_{int} = 0.8$  m, and  $R_{int} = 1.0$  m. It should be noted that  $R_{int} = 1.0$  m corresponds to the unbaffled tank. The

obtained frequencies are presented in Table 2. The number of elements along the meridian and along the radius of the free surface were chosen as  $N_G = 200$  and  $N_0 = 100$ , respectively.  
*Table 2. Natural frequencies of non-axisymmetric ( $l = 1$ ) oscillations of the liquid free surface in a spherical tank with baffles of different diameters*

| $k$ | $R_{int} = 0.2\text{m}$ | $R_{int} = 0.8\text{m}$ | $R_{int} = 1.0\text{m}$ |
|-----|-------------------------|-------------------------|-------------------------|
| 1   | 1.42333                 | 2.093621                | 2.123422                |
| 2   | 5.84067                 | 5.942312                | 5.980714                |
| 3   | 9.45674                 | 9.432785                | 9.478943                |

It should be noted that for  $R_{int} = 1.0\text{ m}$ , the data presented in Table 2, correspond to the shell without a baffle.

As a next step, the required number of terms in series (3) to achieve the desired accuracy was determined. It was found that, to reach an accuracy of  $\varepsilon = 10^{-3}$ , three terms in series (3) are sufficient. We consider a spherical shell of radius  $R = 1\text{ m}$ , both without a baffle and with the baffle installed at the height  $H_1 = 1.0\text{ m}$ , with a filling level of  $H = 1.4\text{ m}$ , subjected to horizontal and vertical excitations; that is, we assume that

$$a_h(t) = a_0 \cos \omega_0 t, \quad a_v(t) = a_1 \cos \omega_1 t. \quad (9)$$

For the numerical analysis, the values of the loading parameters (9) listed in Table 3 are adopted.  
*Table 3. Loading parameters*

| Loading case | $a_h$ | $a_v$ | $\omega_h$ | $\omega_v$ |
|--------------|-------|-------|------------|------------|
| A            | 0.0   | 1.0   | 1.0        | 4.2468     |
| B            | 0.5   | 1.0   | 1.0        | 3.1234     |
| C            | 0.5   | 1.0   | 1.0        | 1.1234     |
| D            | 0.5   | 1.0   | 1.0        | 3.7453     |

All load cases whose parameters are given in Table 6.3, lead to unbounded growth of the free surface amplitude over time. This phenomenon is not physically admissible; such vibrations arise due to assumptions that simplify the real physical behavior. For more substantiated conclusions, it is necessary to employ the theory of viscous fluid motion or to consider nonlinear formulations within the theory of potential flow of an ideal incompressible fluid. However, the solutions obtained within the framework of linear theory make it possible to provide a preliminary assessment of the stability of fluid motion in shells and to tune the system away from undesirable resonant frequencies.

Figures 2 - 5 present such results corresponding to the loss of stability of fluid motion in the tank. Curves (black) labeled 1 correspond to fluid motion in a tank without a baffle, while curves labeled 2 (blue) correspond to the cases with the baffle installed at the level  $H_1 = 1.0\text{ m}$ . Filling level in all cases was equal to 1.4m.

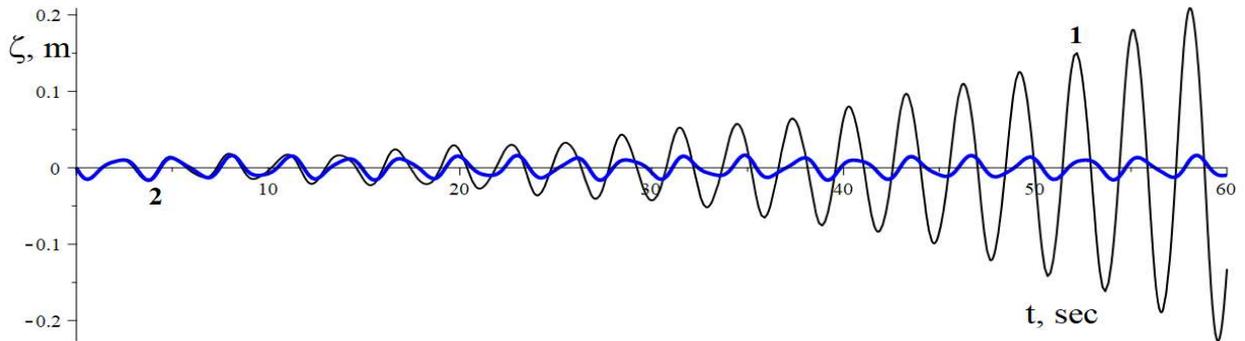


Figure 2. Free surface elevation with and without baffles, Case A.

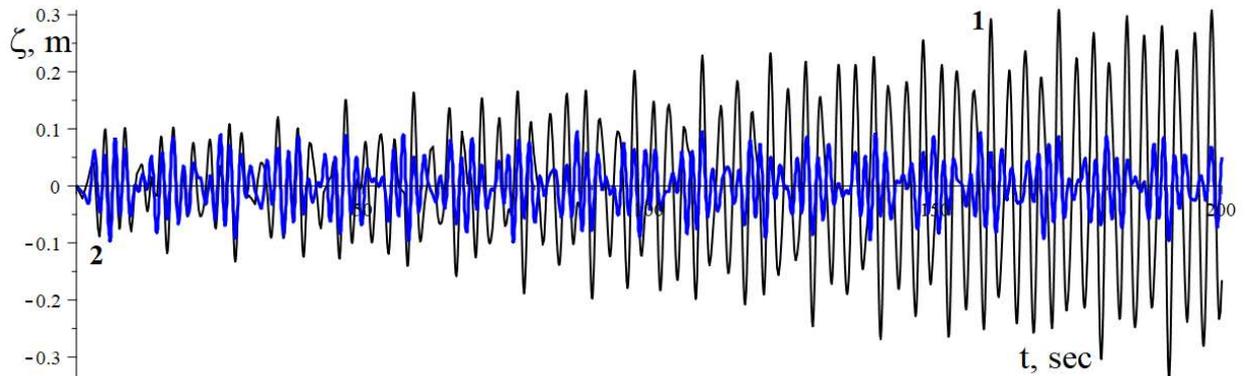


Figure 3. Free surface elevation with and without baffles, Case B.

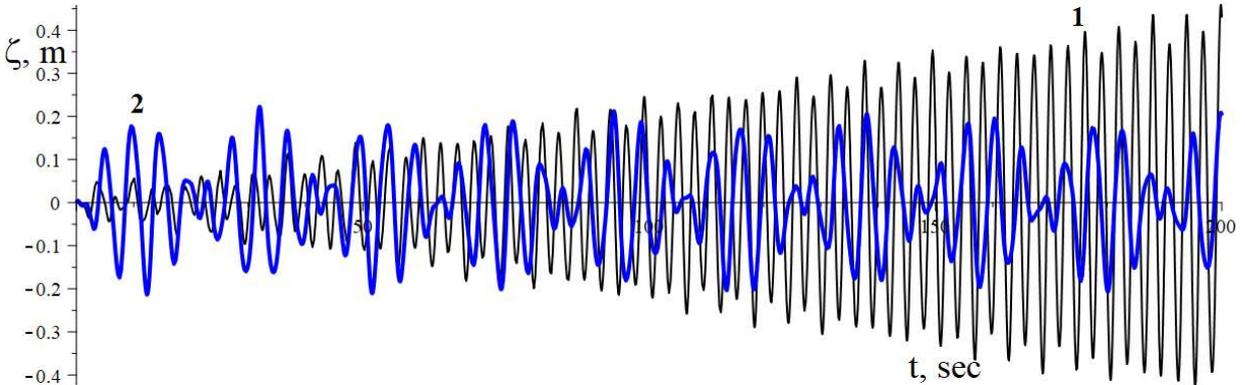


Figure 3. Free surface elevation with and without baffles, Case C.

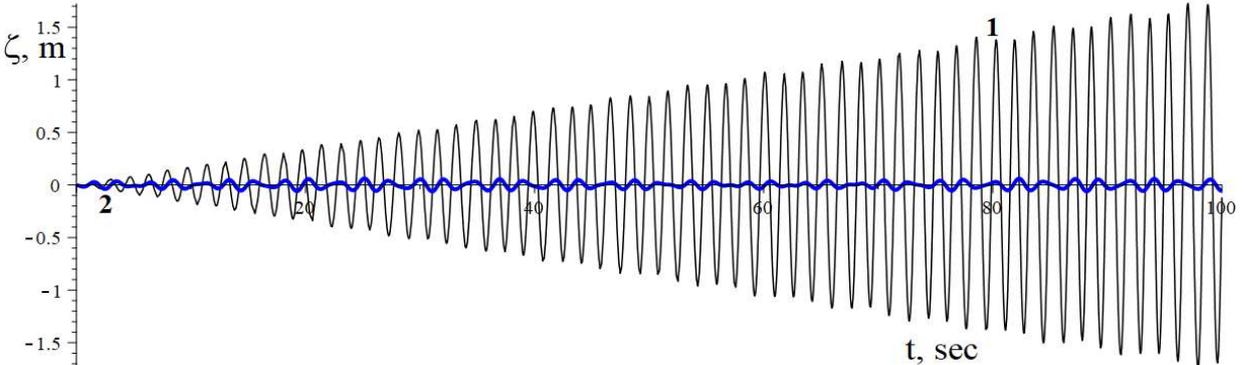


Figure 4. Free surface elevation with and without baffles, Case D.

Figure 2 shows the time-history of the free surface elevation in the case of parametric resonance, when the frequency of vertical excitation is equal to twice the first fundamental natural frequency of liquid oscillations in a rigid tank, corresponding to the first harmonic. In this case, exponential growth of the amplitude is observed (Case A).

Figures 3 and 4 illustrate the time-histories of the free surface elevation in the cases of so-called sub-resonances, when the sum or the difference of the vertical and horizontal excitation frequencies equals the first fundamental frequency.

In Figure 3, the black curve labelled 1, corresponds to the loading case in which the difference between the vertical and horizontal excitation frequencies equals the first fundamental natural frequency (Case B), whereas in Figure 4, the curve labelled 1, corresponds to the loading case in which the sum of the vertical and horizontal excitation frequencies equals the first fundamental natural frequency (Case C).

Figure 6.4 presents data corresponding to liquid oscillations in the presence of resonance, which occurs when the horizontal excitation frequency coincides with the first fundamental frequency of the zero harmonic (Case D)

The obtained results will be useful for the design of rocket fuel tanks and containers for storage dangerous liquids.

## 5. Conclusions and Future Research

This work develops and implements an efficient numerical approach for investigating the oscillations and stability of fluid motion in spherical tanks partially filled with liquid. Based on potential flow theory and the Boundary Element Method, spectral boundary value problems were solved to determine the fundamental frequencies and mode shapes of liquid oscillations, both in tanks without baffles and in tanks with horizontal baffles. These modes are used as basis functions for solving the forced vibration problems. The dynamic problem was reduced to the system of ordinary Mathieu-type differential equations, allowing the study of stability regions of fluid motion in spherical tanks under combined excitations.

Numerical results were obtained characterizing the occurrence of parametric resonance, sub-resonances, and resonance regimes caused by horizontal acceleration. The proposed approach can be used for virtual testing of spherical tanks and for analyzing fluid behavior during the design and operation of tanks and fuel containers in aerospace applications. Future research will focus on studying these resonance phenomena within refined formulations.

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