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The problem of covering the fields by the circles in the task of optimization of observation points for ground video monitoring systems of forest fires

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Abstract. The problem of covering the area by circles, mathematical model of the coating offers a new coverage criteria based on which analytically describes the range of permissible solutions of the problem. Based on the analysis of the properties of the model, it is shown that the solution of the problem can be reduced to the solution of problems of nonlinear programming sequence.

At the moment, there are systems of video monitoring forest land. An important class of geometric design problems is problems of irregular covering the field by geometric objects, as well as regular. In the problems of covering it is set up a claim that all points of the field were covered by geometric objects, while the conditions of non-intersection of objects between themselves and their placement in the field may be violated. One problem with the design of terrestrial video monitoring systems is to optimize the placement of observation points. optimal placement of towers problem can be formulated as a coating task.

An approach to the placement of towers terrestrial video monitoring of forest fires, the main stage of which is set forth search method local extremum in the problem coverage area circles of varying radius.

There is build a mathematical model to optimize the placement of variable radius circle and on its basis – the development of methods of solution and proposed an approach to obtaining a local extremum of covering problem.

Key words: circle, coverage criteria, optimization, nonlinear programming.

INTRODUCTION

One of the approaches to the early detection of forest fires is a monitored [1], as space [2], and ground [3, 4]. Satellite monitoring allows you to quickly identify pockets of fires on forest area of more than (6 - 8) hectares in remote areas with a high update rate and wide coverage area of observation. In this case information obtained remotely, allows not only to analyze the current situation with forest fires, but also to further analyze the dynamics of fire [5–7]. For the detection of fire outbreaks, forest areas are smaller local terrestrial methods using watchtowers and match different designs, industrial video systems. The monitoring data of different levels (ground and space) constitute a single architecture information layers of geographic information systems for monitoring forest fire.

At the moment, there are systems of video monitoring forest land [8–12]. The simplest system [8] allows on the basis of the color image to increase the efficiency, quality and identification of fires in the forests. The monitoring system [9] based on network principle. Video cameras transmit information by radio to a single control point. In the Pskov region tested the system [10], based on forest monitoring with the help of video equipment. The German company offers their Fire Watch developed a system using equipment company IQ wireless [11].

In 2008, the Nizhny Novgorod company "Remote control system" developed an innovative "Forest Watch" forest monitoring system for the early detection of forest fires and determine their origin. [12] "Forest Watch" operates on the basis of modern technologies: IP-CCTV, mobile applications, geographic information systems (GIS), Internet applications and "computer vision".

One problem with the design of terrestrial video monitoring systems is to optimize the placement of observation points. optimal placement of towers problem can be formulated as a coating task. These problems are referred to a class of optimization geometric design problems [13], which solution as well as the development of their methods is important. This class of problems include the problem of optimal material cutting (both regular and irregular), the problems of building the optimal ways and linking networks, coverage, partition, some scheduling problems, and others [13–19].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIOS

An important class of geometric design problems are problems of irregular covering the field by geometric objects [20], as well as regular [20–24]. In the problems of covering it is set up a claim that all points of the field were covered by geometric objects, while the conditions of non-intersection of objects between themselves and their placement in the field may be violated. Results and detailed reviews on given researches are in [25–27]. The problems of single covering a limited area by N-circles (such as in a Euclidean metric, and in some other metrics) is also known as the problem of N-centers. For the problem of N-centers in different metrics it is offered a variety of heuristics and algorithms using Voronoy's regions [28].

Problems of coverage are models of many practical problems. In [29] it is set and solved the problem of interaction between militarized security subdivisions of the railroad and fire-rescue units, which is reduced to covering the area by the circles of different radii. In [30-32] it is set the problem of the placement optimization for observation points, which arises when designing ground video monitoring systems. The problem is reduced to the problem of covering the area by the circles of variable radii, the value of which depends on the class of fire danger of covered area and its relief. Thus, the important practical problems require to develop the methods of covering the fields by circles of variable radius.

OBJECTIVES

The purpose the article is to build a mathematical model to optimize the placement of variable radius circle and on its basis - the development of methods of solution. We propose an approach to obtaining a local extremum of covering problem.

THE MAIN RESULTS OF THE RESEARCH

There is a polygon P, defined by a set of vertices p_k, $k=1,2,...,n$ and a set of circles C_i, $i=1,2,...,N$, with varying radius $r_i < r$ and centers $v_i=(x_i, y_i)$. Suppose $u=(v_1, r_1, ..., v_N, r_N)$ – vector of variables, $F(u)$ - the objective function,

$$\Xi(u)=\bigcup_{i=1}^{N} C_i(u_i, r_i).$$

By definition $\Xi(u)$ – coverage of polygon P if:

$$P \subset \Xi(u) \Leftrightarrow \Xi'(u) \bigcap P = \varnothing,$$

where: $\Xi'(u)=R^2 \setminus \operatorname{int} \Xi(u)$.

Note 1. In this study, we consider only such coverings, which met the following conditions

$$\operatorname{int} C_i \not\subset \Xi_i(u),$$

$$\Xi_i(u)=\bigcup_{j=1}^{i-1} C_j(v_j, r_j) \bigcup_{k=i+1}^{N} C_k(v_k, r_k) \text{ and}$$

$$\operatorname{int} C_i(v_i, r_i) \bigcap \operatorname{int} P \neq \varnothing,\ i=1,2,...,N.$$

Problem of circular coverage of polygon. The start point – vector

$$u^0=(v_1^0, r_1^0, ..., v_N^0, r_N^0),$$

where: $\Xi(u^0)$ covers a polygon P.

The task – to determine the vector

$$u^*=(v_1^*, r_1^*, ..., v_N^*, r_N^*),$$

in which $F(u)$ reaches the extreme and $\Xi(u^*)$ is coverage of a polygon P.

The problem of covering polygon by circles. Mathematical model of the problem of circular coverage can be represented as follows:

$$\underset{u \in W}{extr}\, F(u), \tag{1}$$

where:

$$W=\{u \in R^{3N} : \Xi'(u) \bigcap P = \varnothing\} \tag{2}$$

As a criterion of covering for a fixed u can be used phi-function method [24]:

$$\Xi'(u) \bigcap P = \varnothing \Leftrightarrow \Phi^{\Xi' P} \geq 0,$$

where: $\Phi^{\Xi' P}$ – phi-function of objects $\Xi'(u)$ and P [29].

Since the description of admitted region of the form (2) in an analytical form is extremely difficult theoretical problem and requires significant computational cost, in this study we propose the coverage criteria based on the following statement.

Statement. In order $\Xi(u)$ to cover the polygon P, it is necessary and sufficient that the vector $u=(v_1, r_1, ..., v_N, r_N)$ satisfies the condition:

1) $\forall p_k\ k \in I_n$, exists such circle C_i, that

$$p_k \in C_i,\ C_i \not\subset P;$$

2) if there is a point

$$t \in frC_i \bigcap frP,\ C_i \not\subset P,$$

then there are a circle C_j and a point $v_{ij}, i \neq j$, such that

$$v_{ij} \in C_i,\ v_{ij} \in C_j,\ v_{ij} \in R^2 \setminus \operatorname{int} P;$$

3) if there is a point $t = frC_i \bigcap frC_j$ and where in $t \in frP$, there is C_s and a point $v_{ijs}, s \neq i, s \neq j$, such that

$$v_{ijs} \in C_i,\ v_{ijs} \in C_j, v_{ijs} \in C_s;$$

4) if there is a point $t_q = frC_i \bigcap frC_j$ and $t_q \in \operatorname{int} P$, $q = 1{,}2, i \neq j$, then there is a circle C_s and a point $v_{ijsq}, s \neq i, s \neq j$, such, that

$$v_{ijsq} \in C_i,\ v_{ijsq} \in C_j, v_{ijsq} \in C_{sq}.$$

On the Fig. 1 there is an example of a polygon coverage by the set of circles with the points of type $v_{ij}, v_{ijs}, v_{ijs1}, v_{ijs2}$.

With this in mind, the inequalities describing the admitted region of the problem based on the information about the start point can be written as:

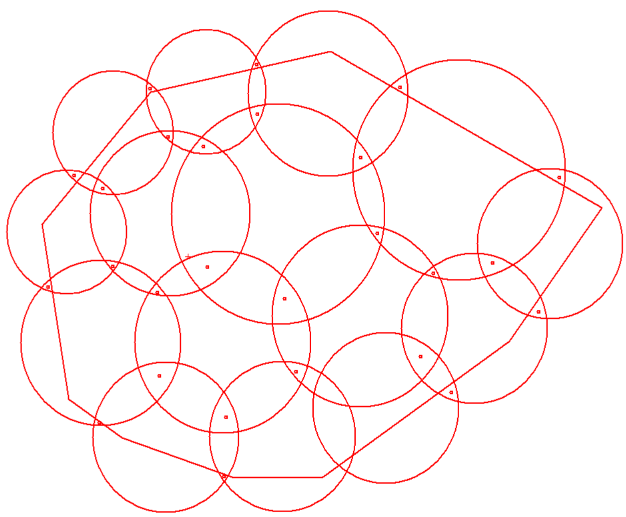


Fig.1. Example of coverage by circles with system of auxiliary points

1) for the vertices of the polygon $\forall p_k\ k \in I_n$, and the corresponding circles C_i, $p_k \in C_i$, $C_i \not\subset P$, inequality:

$$(x_{i_1} - x_k)^2 + (y_{i_1} - y_k)^2 \leq r_{i_1}^2,$$

$$(i_1, k) \in \Xi_1, \tag{3}$$

2) for points $t \in frC_i \bigcap frP$, circles $C_i \not\subset P$ and the corresponding circles C_j is the inequalities system in the form:

$$\begin{cases} (x_{i_2} - x_{i_2 j_2})^2 + (y_{i_2 i_2} - y_{i_2 j_2})^2 \leq r_{i_2}^2 \\ (x_{j_2} - x_{i_2 j_2})^2 + (y_{j_2} - y_{i_2 j_2})^2 \leq r_{j_2}^2, \\ f_{i_2 j_2}(x_{i_2 j_2}, y_{i_2 j_2}) \geq 0 \end{cases}$$

$$(i_2, j_2) \in \Xi_2, \tag{4}$$

where: $f_{i_2 j_2}(x_{i_2 j_2}, y_{i_2 j_2}) \geq 0$ – the membership function of the set $R^2 \setminus \operatorname{int} P$ of points $v_{i_2 j_2}$ (maximum of k linear functions);

3) for points $t = frC_i \bigcap frC_j$, $t \in frP$ and the corresponding circles C_s – the inequalities system in the form:

$$\begin{cases} (x_{i_3} - x_{i_3 j_3 s})^2 + (y_{i_3} - y_{i_3 j_3 s})^2 \leq r_{i_3}^2 \\ (x_{j_3} - x_{i_3 j_3 s})^2 + (y_{j_3} - y_{i_3 j_3 s})^2 \leq r_{j_3}^2, \\ (x_s - x_{i_3 j_3 s})^2 + (y_s - y_{i_3 j_3 s})^2 \leq r_s^2 \end{cases}$$

$$(i_3, j_3, s) \in \Xi_3, \tag{5}$$

4) for points $t_q = frC_i \bigcap frC_j$ and $t_q \in \operatorname{int} P$, $q = 1{,}2, i \neq j$ and the corresponding circles C_{sq} – the inequalities system in the form:

$$\begin{cases} (x_{i_4} - x_{i_4 j_4 s_1})^2 + (y_{i_4} - y_{i_4 j_4 s_1})^2 \leq r_{i_4}^2 \\ (x_{j_4} - x_{i_4 j_4 s_1})^2 + (y_{j_4} - y_{i_4 j_4 s_1})^2 \leq r_{j_4}^2 \\ (x_{s_1} - x_{i_4 j_4 s_1})^2 + (y_{s_1} - y_{i_4 j_4 s_1})^2 \leq r_{s_1}^2 \end{cases}$$

$$(i_4, j_4, s_1) \in \Xi_{41}, \tag{6}$$

and

$$\begin{cases} (x_{i_4} - x_{i_4 j_4 s_2})^2 + (y_{i_4} - y_{i_4 j_4 s_2})^2 \leq r_{i_4}^2 \\ (x_{j_4} - x_{i_4 j_4 s_2})^2 + (y_{j_4} - y_{i_4 j_4 s_2})^2 \leq r_{j_4}^2, \\ (x_{s_2} - x_{i_4 j_4 s_2})^2 + (y_{s_2} - y_{i_4 j_4 s_2})^2 \leq r_{s_2}^2 \end{cases}$$

$$(i_4, j_4, s_2) \in \Xi_{42}. \tag{7}$$

Thus, the mathematical model of the problem coverage area can be formulated as a problem of local optimization:

$$\min_{u \in W \subset R^\delta} F(u) \tag{8}$$

$$u=(u_1,u_2,\dots,u_N,v_{i_2j_2},(i_2,j_2)\in\Xi_2,$$
$$v_{i_3j_3s},(i_3,j_3,s)\in\Xi_3,v_{i_4j_4s_1},(i_4,j_4,s_1)\in\Xi_{41},$$
$$v_{i_4j_4s_2},(i_4,j_4,s_2)\in\Xi_{42})$$

in the area of feasible solutions W, taking into account the limitations of the system (3)-(7). Here:

$$\sigma=3N+2|\Xi_2|+2|\Xi_3|+2|\Xi_{41}|+2|\Xi_{42}|,$$

Ξ - cardinality of the set Ξ, Ξ_1 - set of indexes of pairs of numbers of vertices polygon and circles, satisfying the conditions of paragraph 1 of the criterion of the existence of covering the polygon by the circles, Ξ_2 - set of indexes of pairs of numbers of vertices polygon and circles, satisfying the conditions of paragraph 2 of the criterion of the existence of covering the polygon by the circles, Ξ_3 is set of indexes of pairs of numbers of vertices polygon and circles, satisfying the conditions of paragraph 3 of the criterion of the existence of covering the polygon by the circles, Ξ_{4q}, $q=1,2$ - set of indexes of triples of numbers of vertices polygon and circles, satisfying the conditions of paragraph 4 of the criterion of the existence of covering the polygon by the circles,

$$u_i=(x_i,y_i,r_i)=(v_i,r_i),\ i=1,2,\dots,N;$$

$v_{i_2j_2}=(x_{i_2j_2},y_{i_2j_2})$, $v_{i_3j_3s}=(x_{i_3j_3s},y_{i_3j_3s})$, $v_{i_4j_4s_q}=(x_{i_4j_4s_q},\ y_{i_4j_4s_q})$ is the coordinates of auxiliary points that are used to formalize the terms of covering.

Search of local optimal solution is performed using nonlinear of package of open source IPOPT. It should be noted that at the last iteration of the algorithm point of local extremum in the subregion is the points of local extremum of the initial problem.

Given a starting point - a vector

$$u^0=(u_1^0,r_1^0,\dots,u_N^0,r_N^0),$$

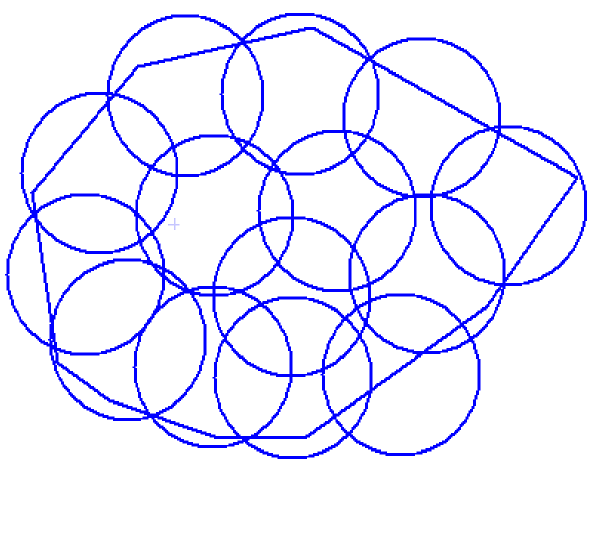
in which $F(x_1^0,y_1^0,r_1^0,\dots,x_N^0,y_N^0,r_N^0)$ is a covering P (Fig. 2a).

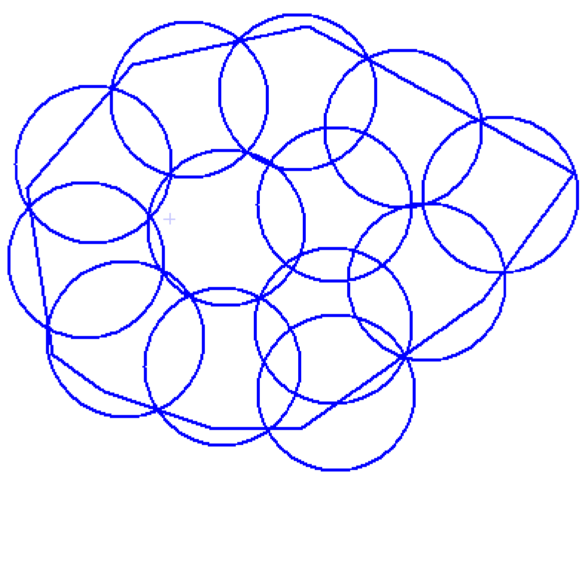
It is necessary to define a vector $u^*=(x_1^*,y_1^*,r_1^*,\dots,x_N^*,y_N^*,r_N^*)$, in which $F(u)$ is a covering P, and the radii of the circles have reached the minimum value (at zero - the circle is removed). On Fig.2b is an example of problem solving.

An algorithm for solving the problem of location of observation terrestrial systems video monitoring forest fires [30], the main stages of which are as follows:

1. Construction start permissible coverage and corresponding function tables gridded values based on the analysis of the relief and fire safety zones.
2. Selection on the basis of the analysis of the tabulated values in the vicinity of each of the towers interpolation areas for which interpolation polynomial satisfies the constraints on the maximum permissible error, the transformation of the selected area to the square of the size of the unit (if appropriate transformation of the detector coordinates) and the construction of the coefficients of interpolating polynomials.
3. Generation of a system of nonlinear inequalities describing the range of permissible solutions of the problem.
4. Search locally optimal solutions generated subdomain of feasible solutions for the methods described above. Contact coordinate transformation fire detectors. If you receive improved value of the objective function, then a transition to the second point, otherwise the completion of the algorithm.

Conclusion. An approach to the placement of towers terrestrial video monitoring of forest fires, the main stage of which is set forth search method local extremum in the problem coverage area circles of varying radius.





a) b)

Fig. 2. Covering of polygon by circles: a) the initial covering of area; b) optimization result

CONCLUSIONS

The constructed model of covering the polygon by the circles of variable radii and a method for obtaining local extremum is basic for a wide range of practical problems, in particular for the problem of locating points of video surveillance, which arises when designing ground video monitoring systems.

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