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GEOMETRICAL MODELING OF THE UNFOLDING OF SPATIAL ROD STRUCTURES, SIMILAR TO THE FOUR-LINK PENDULUM, IN WEIGHTLESSNESS

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Продовжено дослідження геометричних моделей розкриття в умовах невагомості орбітальних стрижневих конструкцій, елементи яких поєднані подібно чотириланковому маятнику [21–24]. Переміщення ланок конструкції відбуваються завдяки дії імпульсів піротехнічних двигунів на кінцеві точки ланок. Опис руху одержаного інерційного розкриття стрижневої конструкції виконано за допомогою рівняння Лагранжа другого роду, і, зважаючи на умови невагомості, побудованого з використанням лише кінетичної енергії системи.

Актуальність теми визначається необхідністю удосконалення та дослідження нових технологічних схем розкриття каркасів космічних інфраструктур. У тому числі каркасів параболічних антен, елементами яких є сім'я однакових співфокусних парабол, одержаних обертанням з певним кутовим кроком навколо спільної осі. Крім того, цікавими мають бути нові технології виконання монтажних робіт на орбіті з використанням конструкцій механічних захватів (типу «руки робота»), розташованих зовні космічних апаратів.

На основі інерційного розкриття чотириланкових стрижневих конструкцій розроблено схеми дії маніпуляторів для захвату циліндричних тіл, осі яких розташовано паралельно або перпендикулярно відносно поверхні космічного апарату. Визначено параметри та початкові умови запуску руху чотириланкової стрижневої конструкції з метою одержання необхідного розташування ланок. Показано, що для впроваджень варіантів інерційного розкриття необхідно застосувати комплект уніфікованих піротехнічних пристроїв, величини імпульсів яких визначаються координатами вектора U'={0.1, 1.9, 1.3, 2.5} умовних одиниць. Побудовано графіки зміни у часі функцій значень кутів як узагальнених координат, а також перших та других похідних цих функцій. В результаті надано оцінки силовим характеристикам системи в момент гальмування (зупинки) процесу розкриття.

Результати призначено для геометричного моделювання варіантів розкриття чотириланкових стрижневих конструкцій в умовах невагомості. Наприклад, каркасів для орбітальних інфраструктур, а також механічних маніпуляторів для захвату космічних об’єктів

Ключові слова: стрижнева конструкція, процес розкриття у космосі, маніпулятор для захвату тіл, рівняння Лагранжа другого роду

1. Introduction

Problems on the dynamics of pendulum systems have been a traditional subject of research by technical mechanics. Such problems were studied even by Galileo who discovered the isochronism in small fluctuations of a body with a fixed horizontal axis and who formalized the phenomenon of oscillations in the form of a mathematical model of the pendulum. Pendulum mechanical systems are very varied. These include pendulums with a vibratory point of suspension, pendulums with a periodically adjustable length, combined pendulums, pendulums that include elastic elements, etc. The problem on flat oscillations of a satellite in elliptical orbit could also be considered as an example of the pendulum system. The peculiarity of specific problems on the dynamics of pendulums is a combination of simplicity of physical statement and the complexity of solutions. The need to obtain such solutions, as well as mechanical interpretations, defines the relevance of problems in modern mechanics.

One such problem addresses the performance of pendulum systems in weightlessness. Here, mechanical interpretation could be associated with the oscillation (transformation) of elements in a multi-link pendulum aimed to provide the rod structure with the required shape after its delivery into the orbit in a folded form. Paper [1] stressed that the majority of spacecraft are the spatially developed mechanical systems by their mechanical essence, with a varied configuration intended to work in space under conditions close to weightlessness. Models of dynamics are typically represented by systems of differential equations of motion, and mathematical modelling implies numerical solutions to these equations or analytical interpretations of possible solutions. The special features in the calculation of the unfolding of large-size structures with various configurations are described in paper [2]. Examples of the space objects that are transformed in space are given in articles [3, 4].

In order to avoid ambiguity, we shall hereafter refer to the rectilinear rods that are interconnected similar to the elements of a multi-link pendulum and are folded during delivery similar to a household centimeter. The rods are made of ultra-light materials such as polymeric composite materials based on epoxy resin, reinforced with graphite or kevlar fibers. When delivered into orbit, the planned spatial shape of the structure is acquired through the transformation of the rods using the mechanical operation of *unfolding*. The unfolding of the four-link rod structure is considered to be an analog to the "oscillation" of a four-link pendulum in weightlessness. Note that the application of the term "pendulum" in the case of weightlessness is not appropriate. Therefore, we shall hereafter use the term "a four-link rod structure (or system)".

Managing the unfolding of large-size structures in space is a complex scientific and technical challenge of mechanics, which has no analogues in the ground-based equipment. When implementing any scheme for the unfolding of a rod structure in weightlessness there is the task to choose technical devices to trigger (activate) the process of unfolding. In practice, a one-time unfolding is the most likely when the rod structure acquires a geometric shape and gets fixed immediately after its delivery into orbit. Electric motors or other technical devices (elastic elements and metals with "thermal memory") would then make an extra (and not cheap) ballast upon the unfolding and fixing the structure.

An alternative to the traditional motors for unfolding could be the pyrotechnic pulse jet engines (a pyropack type). Paper [5] gives basic information about the pyrotechnic devices for use in space infrastructures. The advantages of these devices include small weight and low cost, as well as a capability to configure the required and precalculated magnitude of momentum. They could withstand overload in the process of delivery into orbit. Pyrotechnic pulse jet engines do not need decompression and testing prior to the unfolding of the structure. It is important that the activity could be launched remotely (without wires) using radio signals or X-ray radiation. There is no need to supply current and switch operation. All this prompted to consider pyrotechnic pulse engines as promising devices for the unfolding of "pendulum-like" rod structures. In this case, pulse engines must be installed at the endpoints of links in a rod structure. We expect that a relatively cheap technology for unfolding the rod multi-link structures would be acceptable if there is a need to perform a one-time unfolding.

The dynamics of the process of unfolding a structure in the form of a four-link rod system is appropriately explored based on the Lagrange variational principle. There is an issue on adapting the "oscillation" of a four-link pendulum to weightlessness as a basis of the geometrical model for the unfolding of an orbital object. The answer to this question could be found in studies that tackle the application of the Lagrange equations of the second kind for mechanical systems in weightlessness [6]. Formally, it is considered that calculations regarding the transformation of mechanical rod structures in weightlessness over time could be performed using only the concepts of kinetic energies. That is, when constructing the Lagrange equations of the second kind, the potential energy of a conservative mechanical system could be considered to be "close to zero". Upon enabling the oscillations by pyrotechnical pulses, the magnitude of the kinetic energy for a small period of time should remain unchanged. This assumption makes it possible to develop a formal (idealized) approach to the geometrical modeling of unfolding the rod structures – analogs to four-link pendulums. Geometrical modeling implies the recording of computer-animated films that visualize the mutual displacement of links in rod structures in the process of unfolding. In the further research, we plan to record such assumptions. The use of the developed models would help at the design stage to calculate the arrangement and functioning parameters of the structure in general.

Thus, the relevance of the chosen area of research is emphasized by the need to study and implement pulse jet devices as simple and cheap engines for the process of unfolding rod structures of the four-link pendulum type. That would be economically justified when the process of unfolding the structure in orbit is planned to be performed only once. We propose to use as such engines the pulse pyrotechnic jet engines installed at the endpoints of links in a four-link rod structure. Pyrotechnic devices have several advantages in comparison with known techniques for starting the unfolding of a structure. All this testifies to the importance of study into geometrical models of unfolding the rod structures under conditions of weightlessness with pulse engines at the end points of their links.

2. Literature review and problem statement

When devising schemes for unfolding the multi-link structures, there is a need to construct mathematical models that would adequately describe motion. Underlying our studies is the concept of a multi-link pendulum. Paper [7] reports a study into the dynamics of an *N*-link pendulum; article [8] into simulation of oscillations of the pendulum whose elements are geometrical objects. However, papers [7, 8] lack the graphical confirmation of the simulation result. Certain features of the mathematical processor *maple* to simulate the oscillations of a multi-link pendulum were demonstrated in work [9]. Paper [10] addresses the application of the method of separate bodies to model multi-link movable structures of spacecraft. The example of a mechanical system of unfolding solar battery panels wings is used to form the matrix of kinematic relationships that define the kinematics of relative motion of the adjacent bodies in the system. Article [11] proposed a description of the dynamics of solar panels in the process of their unfolding, taking into consideration the elastic properties of the elements. The authors constructed mathematical models of mechanisms for unfolding, rope synchronization, braking and fixing the panels. Paper [12] calculated the oscillations of an *N*-link inverse pendulum. Article [13] proposed a technique to manage a multi-link inverse pendulum in a plane in the vicinity of the set position of equilibrium. Employing the technique, it is possible to drive the pendulum over finite time to the position of equilibrium by a limited momentum applied to the first link. In contrast to a flat pendulum driven by the scalar momentum, control over pendulum with a two-stage hinges in paper [14] is a two-dimensional vector. This circumstance required a modification in the algorithm of building the control. Control is implemented in the form of a feedback that drives the pendulum from the bound of an arbitrary position of equilibrium to the predefined position through a limited momentum applied to the first link. However, papers [13, 14] focused mainly on handling the equilibrium of an inverse pendulum at a cart in the field of earthly gravitation.

The prototype of the technique for unfolding a rod structure, described in this paper, is a rope system of unfolding. It implies control over systems of rods connected similar to a multi-link pendulum, by using a system of ropes, driven by electric motors. In paper [15], authors constructed a mathematical model for the process of unfolding a multi-link structure of the solar cell with a rope system of unfolding. Based on analysis of the kinematic scheme of the system of unfolding, they selected size for the radii of rollers and a transfer ratio between two types of gear mechanisms that ensure the predetermined sequence of links fixation. To study the process of unfolding a solar panel, the authors applied the Lagrange equation of the second kind. Paper [16] addressed mathematical methods for designing a rope system of unfolding a multi-link structure. Based on analysis of the proposed mathematical model, the authors calculated additional angles in the rotation of links caused by the elasticity of ropes in the synchronization system. They determined preliminary tension of ropes in the synchronization system and the structure's parameters to ensure the guaranteed work of the unfolding system when changing the momenta of resistance over a preset range. Papers [17, 18] employed a method of the external approximation for resolving the task on designing a rope system of unfolding a multi-link structure. The possibility of consistent fixation of links (from the last to the first) was investigated depending on the maximum angles of rotation of links controlled by means of rope tension. In this case, there is no need to change the optimal radii of rollers determined in advance.

However, existing schemes for unfolding the rope systems are too complex to implement in the case of large-size links (of the order of tens of meters). This conclusion is based on the necessity to synchronize and switch electric motors in order to adjust the magnitudes of angles in the structures' nodes to provide a multi-link structure with the calculated geometrical shape, which is a separate task.

Paper [19] considered the modeling of unfolding process for multi-link closed space structures with the application of software packages EULER and Adams using the example of a complex antenna contour and a fragment of space reflector. Article [20] reported simulation of the process of unfolding large-size space structures applying modern software complexes using the example of three kinematic schemes for unfolding the solar panels. The general approach to the construction of models of such structures is described. The results of these studies make it possible to simulate the process of unfolding the multi-link space structures, with determining the speed and duration of the unfolding, as well as shapes of intermediate positions.

In papers [21, 22], authors initiated a geometrical model for unfolding a rod structure in the imaginary plane in weightlessness as a multi-link pendulum. They developed a scheme of initiating the oscillations through the influence from a pulse on one of the nodal elements of the pendulum (a model of pulse jet engine). That made it possible to implement the unfolding of a multi-link pendulum by using a single engine, which does not need to synchronize the means of control over the magnitudes of angles at separate nodes of a multi-link structure. Paper [24] investigated a permissible error in the magnitude of the pulse for initiating the unfolding of a multi-link rod structure with the inertial system of unfolding under condition of obtaining the required position of its links. In addition, there is a technique to determine parameters and initial conditions for initiating the oscillations of a two-link rod structure in order to obtain a cyclic trajectory of the endpoint of the second link. Articles [23–25] illustrate some of the provisions that would elucidate the geometrical model of the unfolding of rod structures.

The result of a review of the scientific literature [1–20] is those issues that have not been yet studies by other authors, which allowed us to formulate the following research problem. There is a need to devise techniques:

– to start and stop the motion of multi-link structures in weightlessness through the action of pulse engines on the endpoints of structures' links;

– to determine the required time of fixation (stopping) the unfolding in the event of the emergence of mutual position of links of rod structures in space;

– to estimate strength characteristics of the system at the point of fixation (stopping) of the unfolding.

At the same time, the presence of extended links of the structure should not fundamentally affect the universal implementation of the inertial technique of unfolding.

3. The aim and objectives of the study

The aim of present study is the development of a geometrical model of unfolding under conditions of weightlessness a rod structure similar to a four-link pendulum. To launch the motion of the structure, it is proposed to use the pulse pyrotechnic jet engines mounted at the endpoints of links of the structure.

To accomplish the aim, the following tasks have been set:

– to construct and solve a system of the Lagrange differential equations of the second kind to describe in weightlessness the phases in the motion of elements of the four-link rod structures;

– in order to model the action of pulse engines to the endpoints of links in the rod structure, develop a scheme of starting and stopping its motion;

– by using computer animation, determine the required time point for the fixation (stopping) of the unfolding when there emerges the mutual spatial arrangement of links in the rod structures;

– to build the charts of time-dependent change of the first and second derivatives from the functions of angles' values as the generalized coordinates; and, based on this, estimate strength characteristics of the system at the time of fixating (stopping) the unfolding;

– to give test examples of the geometrical models for forming certain objects in space (the type of a "robot's arm") based on the unfolding of four-link rod structures.

4. Development of a geometrical model of unfolding the rod structure, similar to a four-link pendulum, in weightlessness

4. 1. Explanation of the general scheme for calculating the process of unfolding a rod structure using the Lagrange mechanics

Here are stages in the sequence of activities during geometrical modeling of the process of unfolding the rod structure in the form of a four pendulum in the imaginary plane in weightlessness. It is believed that the first link of the four-link rod structure is attached to a spacecraft. The mass of the device is by orders of magnitude larger than the total weight of the elements of the structure, which is why the node of attachment is considered immovable. Because the mass of the spacecraft is much greater than the mass of the rod structure, its orientation is stabilized. We shall also assume that the rods are made of lightweight and strong carbon fiber, so the entire weight of the rod structure would concentrate in the loads at nodular points.

Stage 1. In the imaginary plane, we select the generalized coordinates of motion of the rod system in the form of angles formed between the links of the structure and a fixed direction. We set parameters for a rod structure, the length of links and the mass of loads. Using the angles and parameters, we describe the formal (virtual) coordinates of the nodal points of links in a four-link rod system.

Stage 2. Using the generalized coordinates and parameters, we derive the Lagrangian of the movable rod system whose description in the case of weightlessness matches the expression for the kinetic energy of the system.

Stage 3. Using the Lagrangian, we construct the Lagrange differential equations of the second kind whose quantity corresponds to the number of functions, the generalized coordinates of the rod system. Typically, analytical expressions for the Lagrange equations of the second kind are too cumbersome. That is why it is advisable to use software packages capable of operating information in the form of analytical expressions. For example, the *maple* software makes it possible to operate with the derived approximate solution to a differential equation similar to a standard function.

Stage 4. Find the approximate solutions to the Lagrange equation of the second kind by using, as conditions, the initial values of angles and instantaneous speeds of change in the values for these angles as the corresponding derivatives. By substituting the derived approximate solutions in the "formal" formulae from stage 2, we obtain descriptions of the actual coordinates of points of links in a four-link rod system.

Stage 5. Based on the obtained time-dependent dependences of the actual coordinates of the endpoints of links, we create a computer animation video recording of the process of unfolding a four-link rod system. Owing to the visualization of the displacement of links in the rod structure, it is possible, in a certain sense, to confirm the adequacy of the obtained solution. To this end, relying on the visual analyzer when watching the frames of the recording, it is necessary to analyze the motion of the rod structure and to identify false solutions in the case of "weird" movements of the links.

Stage 6. The development of the process of unfolding the structure is seen using consecutive frames of computer animation. The result would be the shape (that is, the arrangement of links) of the rod structure that is acceptable for the application. That makes it possible to determine the point in time to stop (block) the unfolding, as well as calculate instantaneous values for the magnitudes of angles (generalized coordinates) that correspond to this point in time. These values define the stop code in order to fixate the unfolding by devices in cylindrical hinges.

Stage 7. Using the obtained approximate solutions to the Lagrange equation of the second kind, we plot the time-dependent change in the magnitudes of angles as the functions of generalized coordinates, as well as the first and second derivatives from these functions. The specified dependences make it possible to calculate the speeds and acceleration of change in the magnitudes of these angles. The above allows us to estimate strength characteristics of the system at a time of stopping and fixing the unfolding. In the case when the values for strength characteristics at nodes turn out too large, we shall proceed to and determine another point in time when the rod structure in the process of stage 6 during unfolding acquires another spatial shape, acceptable for the application.

In the enumerated stages during study into the process of unfolding a four-link rod structure in weightlessness, special role belongs to stages 5 and 6. Underlying these stages is the geometrical modeling of the motion of a mechanical system. Indeed, it would be impossible, without the visualization of separate phases in the unfolding of a structure, to choose the point in time when the rod structure acquires a spatial shape, acceptable for use. Owing to this, at a defined point in time, it is possible to run an analysis of mechanical parameters of the system.

4. 2. Explanation of the principle of unfolding and stopping a four-link rod structure at a specified time

Here is an explanation of the technique of activation and the mechanism of stopping the process of unfolding a four-link rod structure.

Start of the unfolding. We choose an imaginary plane with the Cartesian coordinates Oxy, along which, under conditions of weightlessness, a four-link rod structure must move. It would consist of four weightless non-stretchable rods of length L_1, L_2, L_3 and L_4, connected by nodal cylindrical hinges of mass m_1, m_2, m_3 and m_4. The motion without friction in the cylindrical hinges is ensured by the loads displaced only within the selected imaginary plane. That is, cylindrical hinges in the structure's nodes provide for its unfolding only within the abstract plane, which passes through a motionless point of the structure.

A beginning of the first link of the rod structure coincides with the coordinate origin *O*. A reference direction is the *Oy* axis. The generalized coordinates are angles $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$, formed in the plane between the respective links and the reference direction (Fig. 1).

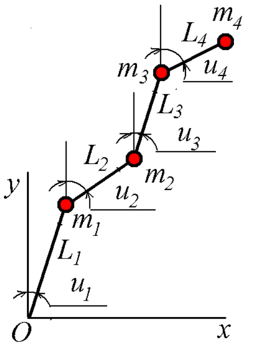


Fig. 1. Schematic of a four-link rod structure

A rod structure is set in motion in weightlessness by selecting the magnitudes of pulses given to each deviation angle [22, 23]. For example, $\mathbf{U'}=\{u_1'(0), u_2'(0), u_3'(0), u_4'(0)\}$ means that the *i*-th load of mass *m* is given the pulse of magnitude $m_i u_i'(0)$, (=1...4). That is, the angles of unfolding u_i(t) are given the initial velocities $u_i'(0)$, (=1...4). Vectors $\boldsymbol{R}_\mathbf{i}$ (=1...4) of the velocity direction setting coincide with the direction of action of pulse engines, directed perpendicular to the corresponding link of the rod structure at endpoints.

Taking into consideration the instantaneous velocities $u_1'(0)$, $u_2'(0)$, $u_3'(0)$ and $u_4'(0)$ given by the jet engines, the rod structure must be subsequently unfolded by inertia. The above explains the term "inertial system of unfolding". Each pulse pyrotechnic engine (a pyropack type) must ensure the magnitude of pulse calculated for it. It is mandatory that jet engines should be fastened so that the actions are directed along a normal to the corresponding link in the plane of unfolding.

To describe the motion of a four-link rod structure, it is necessary to construct and solve a system of the Lagrange equations of the second kind [7–9]. To this end, using the generalized coordinates, we calculate "virtual" coordinates of the nodal points:

$$x_1(t) = L_1 \sin\left(u_1(t)\right);\quad y_1(t) = L_1 \cos\left(u_1(t)\right);$$

$$x_2(t) = x_1(t) + L_2 \sin\left(u_2(t)\right);$$

$$y_2(t) = y_1(t) + L_2 \cos\left(u_2(t)\right);$$

$$x_3(t) = x_2(t) + L_3 \sin\left(u_3(t)\right);$$

$$y_3(t) = y_2(t) + L_3 \cos\left(u_3(t)\right);$$

$$x_4(t) = x_3(t) + L_4 \sin\left(u_4(t)\right);$$

$$y_4(t) = y_3(t) + L_4 \cos\left(u_4(t)\right). \tag{1}$$

In the absence of dissipative forces and taking into consideration the "null" potential energy, description of the unfolding of a rod structure in the imaginary plane will be performed based on a Lagrangian:

$$L = 0{,}5\Big[m_1\left(x_1'^2 + y_1'^2\right) + m_2\left(x_2'^2 + y_2'^2\right) + \\ + m_3\left(x_3'^2 + y_3'^2\right) + m_4\left(x_4'^2 + y_4'^2\right)\Big]. \tag{2}$$

Description of the motion of a four-link rod structure will be obtained in the form of a system of four differential Lagrange equations of the second kind relative to functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ (not given here due to its cumbersome nature). When solving the system of equations, one should consider coordinates of the following vectors: lengths of links of the rod structure: $\mathbf{L}=\{L_1, L_2, L_3, L_4\}$; values of masses of loads (hinges): $\mathbf{m}=\{m_1, m_2, m_3, m_4\}$; values of the initial angles of deviation: $\mathbf{U}=\{u_1(0), u_2(0), u_3(0), u_4(0)\}$, as well as the values of the initial velocities given to the angles of deviation $\mathbf{U'}=\{u_1'(0), u_2'(0), u_3'(0), u_4'(0)\}$. All parameters' values are in conditional magnitudes.

Taking into consideration the respective initial conditions, the system of the Lagrange equations of the second kind was solved by the Runge-Kutta method in the environment of the mathematical software package *maple*; the obtained approximate solutions are denoted with symbols $U_1(t)$, $U_2(t)$, $U_3(t)$ and $U_4(t)$. In the coordinate system *Oxy*, chosen in the plane, by using the obtained solutions, we determine coordinates of the nodal points at time *t*. To this end, we apply expressions (1) to calculate coordinates of the nodes of the rod structure using the generalized coordinates, replacing lowercase letters *u* there with uppercase letters *U* [22].

With respect to the calculated coordinates of the nodes of the rod structure as a function of time, we shall build the frames of computer-animated video of the unfolding process. While watching the process of unfolding, we choose time $t=t_0$ when the unfolding stops and determine parameters for the stop code $\mathbf{U_{STOP}}=\{u_1(t_0), u_2(t_0), u_3(t_0), u_4(t_0)\}$.

Stop of the unfolding. At time t_0, determined based on the computer animation, there is a task on stopping the unfolding. It is believed that in this case the links of the rod structure are arranged in the manner required for using them in the imaginary plane. To stop the motion, it is required to simultaneously employ the special devices built into all cylindrical hinges in the rod structure that could fix the angles between links. For example, it is possible to install a cross-bar into the cylindrical hinge that would be enabled by the action of a pyrotechnic device. The pyrotechnic device must be activated by a signal at a predefined moment t_0 during stopping.

Fig. 2, *a* shows a frame of the launch of the unfolding process (a pyropack is shown in red); Fig. 2, *b* shows a frame of the intermediate position of the unfolding process; Fig. 2, *c* shows a frame of stopping the unfolding of the rod structure using a cylindrical cross-bar (marked in red). If applied once only, there is a possibility to punch through both cylinders of the hinge. The animated video of activating and stopping the process of unfolding could be watched at the Internet site [26].

Crossbars in all cylindrical hinges are triggered simultaneously via a remote control over pyrotechnical devices. A signal to stop the process of unfolding is enabled by software in the form of a "stop-code" at moment t_0 during unfolding, which would ensure the required arrangement of all links in the rod structure.

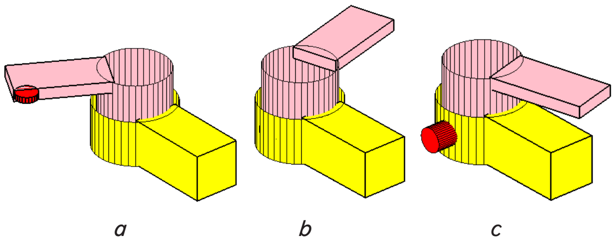


Fig. 2. Video frames of action of pyrotechnic devices: *a* — to activate the motion (pyropack is in red); *b* — to move the rod by inertia; *c* — to stop the process of unfolding using a cylindrical cross-bar

Thus, the functioning of the considered scheme of unfolding a four-link rod structure requires two types of pyrotechnic devices. Devices of the first type are at the endpoints of links in the structure and are intended to start the unfolding. They are characterized by high accuracy in the magnitude of the "start" pulse. These magnitudes might be different depending on the link of the structure. The order to start the unfolding is sent simultaneously to all pyrotechnic devices of the first type. Devices of the second type are installed in cylindrical hinges and are intended to trigger the cross-bars. They are characterized by the magnitude of the pulse that is enough to fixate the cylinders of the hinge, including by punching through. Therefore, control over the considered unfolding scheme of a four-link rod structure implies sending two signals to the pyrotechnic devices – to start and to stop the unfolding.

To prevent excessive loads on the elements of the structure that arise when stopping the unfolding of the structure, rods should be fabricated from ultralight materials. For example, polymeric composite materials based on epoxy resin, reinforced with kevlar fibers.

4. 3. Test examples of geometrical modeling of the unfolding of a four-link rod structure

Here are some test examples of the geometrical modeling of certain orbital objects whose geometric shape is based on unfolding the four-link rod structures. The result of execution of the program is the time-dependent sequence of frames from the animated images of the unfolding of the structure. When watching the video frames, we choose the required arrangement of links in rod structures and fix time t_0, which would correspond to the chosen arrangement. At the same time, we obtain approximate values for the current magnitudes of angles $u_1(t_0)$, $u_2(t_0)$, $u_3(t_0)$ and $u_4(t_0)$ for the chosen time t_0. These values will be used to form a "stop-code" of the unfolding process. The examples are illustrated by axonometric images of the obtained resulting phases of links in a rod structure.

A multi-link rod structure is delivered into orbit in the folded form (visually, it reminds a household centimeter in folded position). That is, the initial position of the set of links in a rod structure takes a "folded" form, and the vector of values of the initial angles of deviations always accepts coordinates $\mathbf{U}=\{\pi/2, -\pi/2, \pi/2, -\pi/2\}$. By using the program developed by *maple,* it is possible, in addition to the displacement of the nodal points, to determine their speeds that makes it possible to build the respective phase trajectories of displacement. The mapped plots for the acceleration of change in the angles of unfolding make it possible to determine strength characteristics in the hinges between the links at a point in time when a rod structure is fixed.

Input parameters. For all the examples we have chosen the same length of links $\mathbf{L}=\{3, 3, 3, 3\}$ and mass of loads $\mathbf{m}=\{1, 1, 1, 1\}$. In addition, the endpoints of links of the mechanisms are to be influenced by a set of the unified pyrotechnic devices whose pulses magnitudes are determined by the coordinates of vector $\mathbf{U}'=\{0.1, 1.9, 1.3, 2.5\}$. Specific implementations in the examples differ by the time for integrating the system of the Lagrange equations of the second kind (that is, duration of the process of unfolding a rod structure). As well as by the coordinates of the vector of a "stop-code" $\mathbf{U}_{\mathbf{STOP}}$ that define the achievement of mutual arrangement of links at a time when the unfolding stops. The values for all parameters are given in conditional magnitudes.

Example 1. From a practical point of view, it is of interest to unfold space parabolic antennas. In this case, the main issue would be the construction of a spatial frame consisting of the family of co-focal parabolas rotated around a common axis. For the sake of correctness, we shall hereafter use the term "quasi-paraboloid". We shall demonstrate the geometrical modeling of the scheme for unfolding the four-link rod structures in order to approximate a shape of the frame of a quasi-paraboloid.

First, we build a quasi-parabola. To approximate the shape of a quasi-parabola by the four-link rod structures, it is necessary that the endpoints of their links are exposed to the action of a set of the unified pyrotechnic devices. Duration of integration of the system of equations is T=1.55.

Fig. 3 shows video frames of the scheme for forming a quasi-parabola, approximated by two four-link rod structures. The values for the coordinates of a "stop-code" vector will be defined by numbers: $\mathbf{U}_{\mathbf{STOP}}=\{0.1642, 0.7710, 1.006, 1.435\}$.

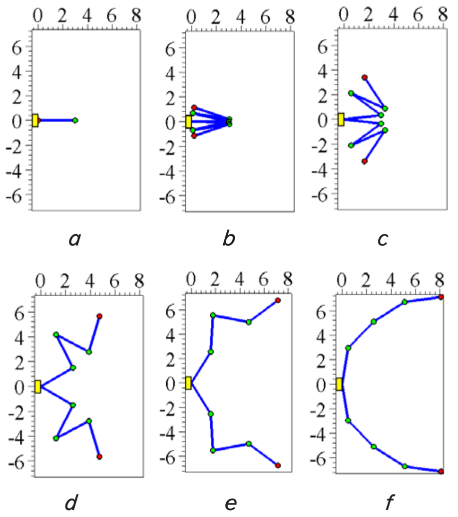


Fig. 3. Video frames of the process of unfolding the scheme of a quasi-parabola: *a* — t=0; *b* — t=0.2; *c* — t=0.5; *d* — t=1; *e* — t=1.2; *f* — t=1.55

Fig. 4 shows the resulting shape of the frame of a quasi-paraboloid, obtained by rotating the quasi-parabola from Fig. 3, *e* around the *Ox* axis

Fig. 5 shows phase trajectories of the generalized coordinates for unfolding a rod structure for example 1. At the final stage of the unfolding, the speeds of the respective nodes will accept the following values: $u_1'(1.55)=-0.45$; $u_2'(1.55)=3.3$; $u_3'(1.55)=-5$; $u_4'(1.55)=1.65$.

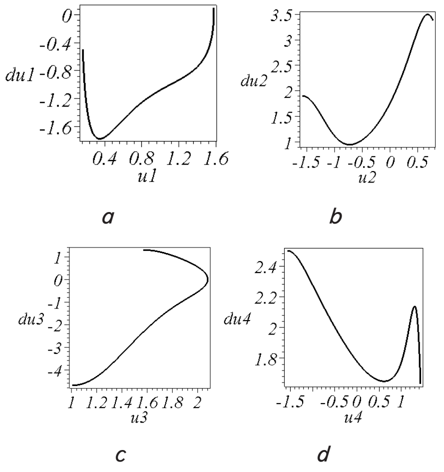


Fig. 5. Phase trajectories of generalized coordinates for example 1: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$

Next, we analyze the forces that act on the nodal elements of a rod structure. Fig. 6 shows the plots of forces acquired from the acceleration chart, multiplied by the respective masses. At the final stage of unfolding, forces in the respective nodes will accept the following values: F_1 (1.55)=20; F_2 (1.55)=–8; F_3 (1.55)=–1; F_4 (1.55)=–14 conventional units.

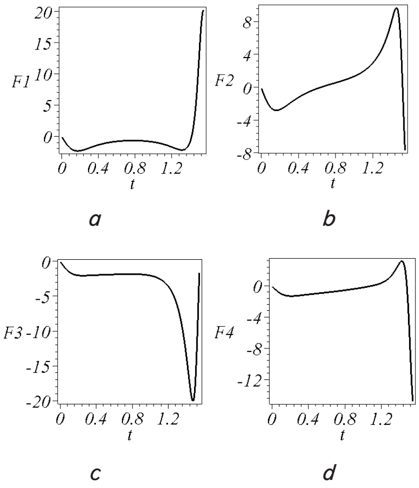


Fig. 6. Plots of forces that influence the nodal elements for example 1: *a* – $F_1(t)$; *b* – $F_2(t)$; *c* – $F_3(t)$; *d* – $F_4(t)$

Example 2. When performing installation operations in orbit, it is required to use the designs of mechanical grips of the "robot's arm" type. For the sake of simplification, we shall consider variants of gripping a cylindrical body. We describe the geometrical modeling of a scheme of the manipulator's action based on the unfolding of four-link rod structures to grip a cylinder, located in parallel to the surface of a spacecraft.

First, we consider the scheme of unfolding two four-link rod structures in order to grip the region of the circle with a radius of R=1.3 with coordinates of the center (0, –4.7). To grip the circle, it is required that the endpoints of links are exposed to a set of the unified pyrotechnic devices. Duration of integration of the system of equations is T=3.14.

Fig. 7 shows consecutive video frames of gripping the circle by two four-link rod structures. The values for the coordinates of a "stop-code" vector will be determined by the following numbers: $\mathbf{U_{STOP}}$={0.5102, 2.306, 0.1527, 6.621}.

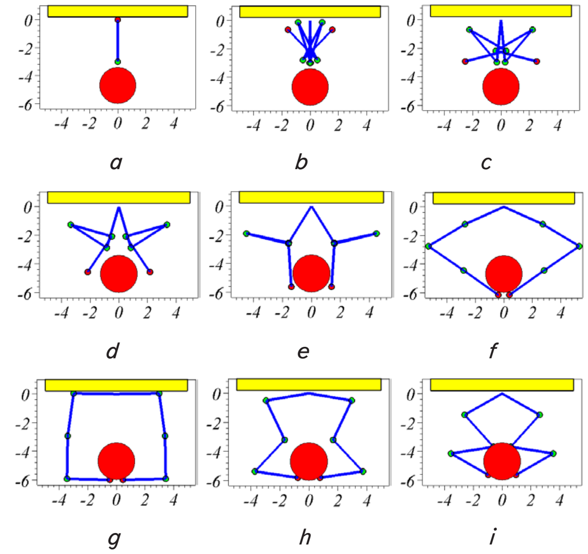


Fig. 7. Video frames of the process of gripping the circle: *a* – t=0; *b* – t=0.3; *c* – t=0.78; *d* – t=1.2; *e* – t=1.52; *f* – t=2; *g* – t=2.5; *h* – t=2.8; *i* – t=3.14

Fig. 8 shows the geometric model of gripping the cylinder, located in parallel to the surface of a spacecraft, based on Fig.7, *f*.

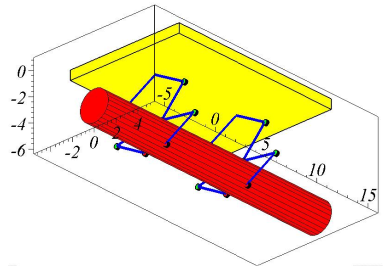


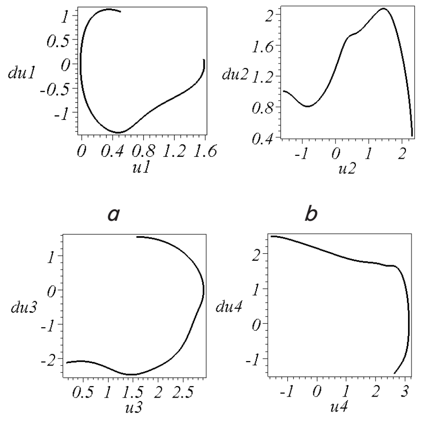
Fig. 8. A pair of rod structures to grip a cylinder in space, located in parallel to the surface of a spacecraft

Fig. 9 shows phase trajectories of the generalized coordinates for unfolding a four-link rod structure for example 2. At the final stage of unfolding, the speeds of the respective nodes will accept the following values: $u_1'(3.14)=1$; $u_2'(3.14)=0.4$; $u_3'(3.14)=-2$; $u_4'(3.14)=-1.5$.

Next, we analyze the forces that act on the nodal elements of a rod structure. Fig. 10 shows the plots of forces obtained from the chart of accelerations, multiplied by the respective masses. At the final stage of unfolding, forces in the respective nodes will accept the following values: F_1 (3,14)=0; F_2 (3,14)=–3.2; F_3 (3,14)=0; F_4 (3,14)=–1.8.

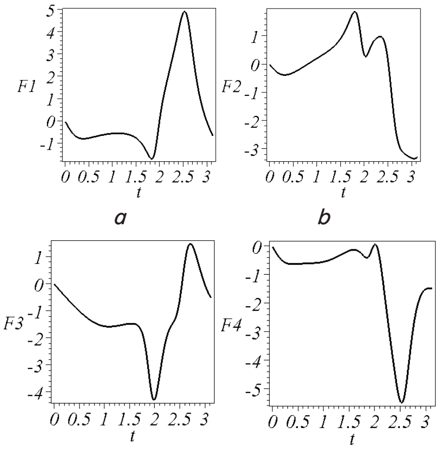
Example 3. Building on the previous example, we consider another simplified variant for gripping a cylindrical body. Namely, we shall describe the geometrical modeling of a scheme of the manipulator's action based on unfolding the four-link rod structures in order to grip a cylinder, located perpendicular to the surface of a spacecraft.

First, we consider the scheme of unfolding the two four-link rod structures in order to arrange the last links in parallel. To do this, the nodal points must be exposed to the action of a set of the unified pyrotechnic devices. Duration of integration of the system of equations must be chosen as T=1.48.



a b c d

Fig. 9. Phase trajectories of generalized coordinates for example 1: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$



a b c d

Fig. 10. Plots of forces that influence the nodal elements for example 2: *a* – $F_1(t)$; *b* – $F_2(t)$; *c* – $F_3(t)$; *d* – $F_4(t)$

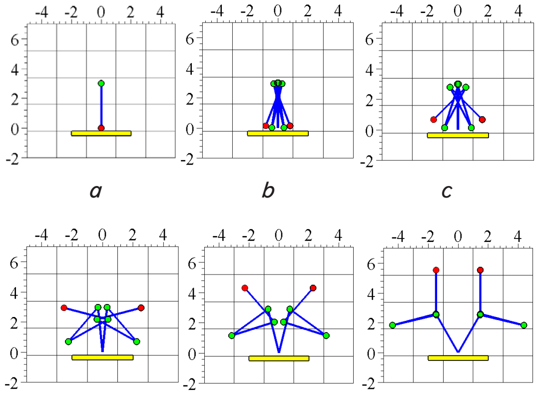
Fig. 11 shows consecutive video frames of deriving a scheme for the arrangement of the last two links in two four-link rod structures with a common fixed point. The values for the coordinates of a "stop-code" vector will be determined by the following numbers: $\mathbf{U}_{\mathbf{STOP}}=\{1.051, -0.256, 2.906, 1.57\}$.

Fig. 12 shows the geometrical model of the scheme for gripping a cylinder with radius R=1.4, located perpendicular to the surface of a spacecraft, based on Fig. 11*f*.

Fig. 13 shows phase trajectories of the generalized coordinates for unfolding a four-link rod structure for example 3. At the final stage of unfolding, the speeds of the respective nodes will accept the following values: $u_1'(1.48)=-0.8$; $u_2'(1.48)=1.05$; $u_3'(1.48)=-0.5$; $u_4'(1.48)=1.78$.

Next, we analyze the forces that act on the nodal elements of a rod structure. Fig. 14 shows the plots of forces obtained from the chart of accelerations, multiplied by the respective masses. At the final stage of unfolding, forces in the respective nodes will accept the following values: F_1 (1.48)=–0.7; F_2 (1.48)=0.8; F_3 (1.48)=–1.5; F_4 (1.48)= =–0.2 conditional units.

Example 4. In some cases, it is necessary to ensure gripping a spatial object but without touching it (because of the cost or safety). To this end, it is advisable to build a grid that would cover the object. We describe the geometrical modelling of a scheme for forming a manipulator-grid based on unfolding the four-link rod structures in order to grip a spatial object.



a b c d e f

Fig. 11. Video frames of ensuring a scheme for arranging the last two links in parallel: *a* – t=0; *b* – t=0.15; *c* – t=0.5; *d* – t=0.8; *e* – t=1.3; *f* – t=1.48

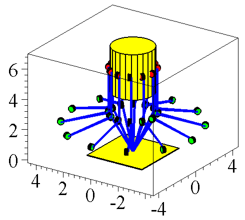
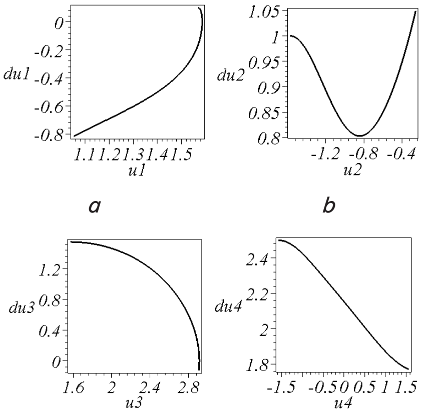


Fig. 12. A rod construction, in which the last links would grip a cylinder in space, located perpendicular to the surface of a spacecraft



a b c d

Fig. 13. Phase trajectories of generalized coordinates for example 3: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$

First, we consider the scheme of unfolding two four-link rod structures with the aim of limiting part of the plane. In order to arrange the last links of the two four-link rod structures in parallel, it is required that the nodal points are exposed to the action of a set of the unified pyrotechnic devices. Duration of integration of the system of equations must be chosen as T=2.38.

Fig. 15 shows consecutive video frames of bringing the arrangement of elements in the two four-link rod structures with a common fixed point to the scheme of a "closed" arrangement. The values for the coordinates of a "stop-code" vector will be determined by the following numbers: $\mathbf{U}_{\mathbf{STOP}}$={0.036, 1.163, 1.871, 3.026}.

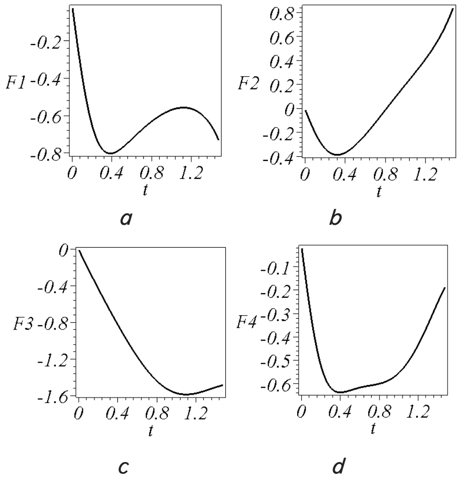


Fig. 14. Plots of forces that influence the nodal elements for example 3: *a* – $F_1(t)$; *b* – $F_2(t)$; *c* – $F_3(t)$; *d* – $F_4(t)$

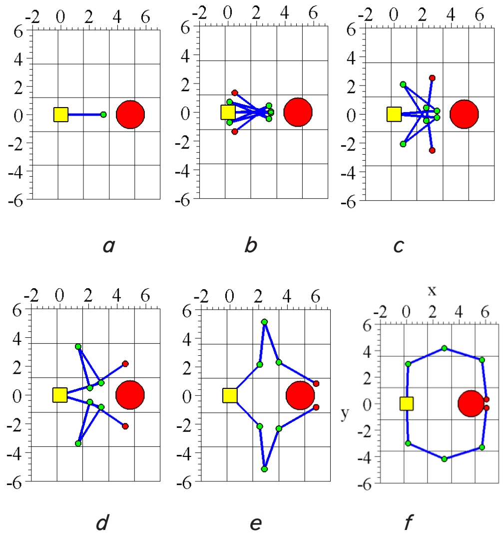


Fig. 15. Video frames of the process of bringing the elements of rod structures to the scheme of a "closed" arrangement: *a* – t=0; *b* – t=0.1; *c* – t=0.5; *d* – t=1.1; *e* – t=1.8; *f* – t=2.38

Fig. 16 shows the geometrical model of gripping a spatial object, based on the scheme from Fig. 15, *f*.

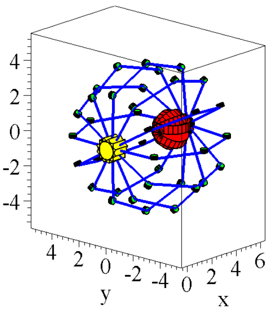


Fig. 16. A rod structure that covers a geometric object in space without a touch

Fig. 17 shows phase trajectories of the generalized coordinates for unfolding a four-link rod structure for example 4. At the final stage of unfolding, the speeds of the respective nodes will accept the following values: $u_1'(2.38)=-0.78$; $u_2'(2.38)=2$; $u_3'(2.38)=-2.2$; $u_4'(2.38)=1.3$.

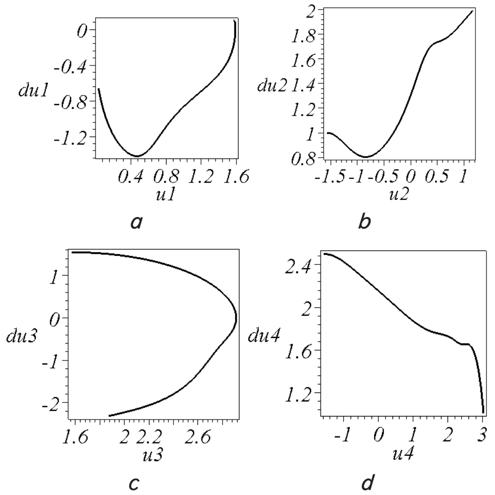


Fig. 17. Phase trajectories of generalized coordinates for example 4: *a* – $u_1(t)$; *b* – $u_2(t)$; *c* – $u_3(t)$; *d* – $u_4(t)$

Next, we analyze the forces that act on the nodal elements of a rod structure. Fig. 18 shows the plots of forces obtained from the chart of accelerations, multiplied by the respective masses. At the final stage of unfolding, forces in the respective nodes will accept the following values: F_1 (2.38)=3.9; F_2 (2.38)=1; F_3 (2.38)=–1.5; F_4 (2.38)=–4.

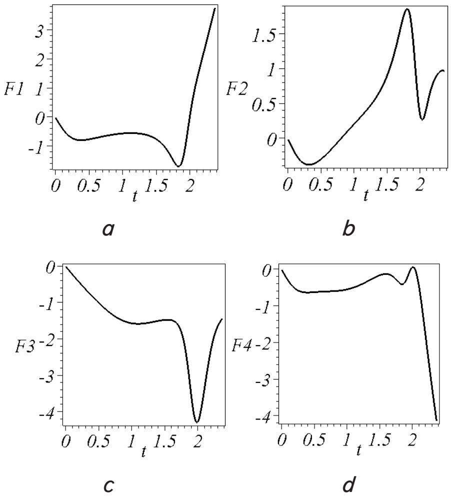


Fig. 18. Plots of forces that influence the nodal elements for example 3: *a* – $F_1(t)$; *b* – $F_2(t)$; *c* – $F_3(t)$; *d* – $F_4(t)$

Therefore, for each of the examples, we calculated and constructed:

– the arrangement of elements in the four-link rod mechanisms in the process of unfolding;

– phase trajectories of functions of generalized coordinates that make it possible to determine the range of change in the values of angles and velocities of the unfolding;

– time-dependent plots of change in the magnitudes of angles as the functions of generalized coordinates, as well as the first and second derivatives from these functions;

– charts of acceleration and strength characteristic of change in the magnitudes of angles as the functions of generalized coordinates;

– variants of unfolding several four-link rod mechanisms with a common fixed point.

For all the examples, we have chosen the same length of links **L**={3, 3, 3, 3} and mass of loads **m**={1, 1, 1, 1}. In this case, it is important that we demonstrated the possibility to use the preliminary prepared set of the standardized pyrotechnic devices whose pulses' magnitudes are defined by the coordinates of vector **U′**={0.1, 1.9, 1.3, 2.5}. Specific implementations in the examples differ by the duration of the unfolding process of a rod structure, as well as by coordinates of the "stop-code" vector $\mathbf{U}_{\mathbf{STOP}}$ when the unfolding stops. The reliability of the results obtained could be confirmed by watching computer-animated videos at the website [26].

5. Discussion of results of modeling the unfolding of rod structures similar to four-link pendulums in weightlessness

In the course of this study we have derived the idealized geometrical models of unfolding the rod structures similar to four-link pendulums. It has been shown that it is advisable to choose, as the engines for the process of unfolding, the pulse pyrotechnic jet engines mounted at the endpoints of links in a rod structure. Such engines could be used when the unfolding of the structure in orbit should be performed only once.

The advantages of pyrotechnic devices underlying the proposed scheme of the unfolding of a multi-link rod structure include:

– low weight and cheapness;

– capability to be adjusted in line with the estimated magnitude of pulse;

– resistance to overloads in the process of delivery into orbit;

– they do not need decompression and testing prior to the unfolding of the structure;

– a possibility to activate the action remotely using radio signals or X-ray radiation.

The results obtained could be explained by the possibility of applying the variational principle of Lagrange to the calculation of mechanical structures using solely the kinetic energy under conditions of weightlessness. That made it possible to use the Lagrange equations of the second kind in order to describe the motion of an analogue of the four-link pendulum in weightlessness, which allowed us to determine the arrangement of links of the rod structure depending on the time of unfolding. The results obtained could also be explained by the possibility to associate the derivatives from initial conditions when solving the Lagrange equations of the second kind with the magnitudes of pulses from the pyrotechnic devices that act at the endpoints of the structure's links. And, finally, the findings could be explained by the possibility to simultaneously launch the pyrotechnic devices remotely at the onset of the unfolding and to simultaneously launch these devices for it to stop.

It is clear that the proposed geometrical model of a four-link rod structure in weightlessness requires further research to bring it closer to an actual structure. It is necessary to take into consideration the moments of inertia of rods when elements of the structure rotate. The opportunities of research into the motion of a pendulum system in weightlessness that remain to be implemented include taking into consideration the unequal lengths of links in a rod structure, as well as irregular masses of nodal elements and links. The development of an illustrative geometrical model of the inertial unfolding of a multi-link rod structure elucidates the use of conditional units for the parameters in the test examples.

A given area of research will be further developed to exploit other variants of multi-link rod structures in which the intermediate nodes of a "parent" structure could act as the starting nodes of the "daughter" multi-link rod structures. This is important when calculating the schemes of unfolding the "star-shaped" geometrical structures, when several rod structures have a common motionless point. The difficulties in the development of research into this area would emerge when trying to solve the inverse problem of arrangement. Specifically, given the preset resulting location of elements inn a rod structure, it is required to define a rational set of parameters for a multi-link pendulum and the initial conditions for its motion, which would ensure the required unfolding. The study conducted is aimed at creating a basis for the calculation of spatial multi-link rod structures whose links would go beyond a certain plane in the process of unfolding.

6. Conclusions

1. The derived solutions to the system of the Lagrange differential equations of the second kind allowed us to describe the movements of a rod structure similar to the four-link pendulum in weightlessness. That made it possible to specify the geometrical model of unfolding the rod structures and watch them under a mode of computer animation.

2. To model the action of a pyrotechnic device, we have developed a scheme for launching the motion of a rod structure through the impact from pulses at the endpoints of its links. It was observed that the pulse of action at a link of the rod structure is numerically equal to the value of the first derivative from a function that describes a change in the magnitude of the corresponding angle as the generalized coordinate. In addition, we have developed a scheme to stop the motion using pyrotechnic devices in the cylindrical hinges between the links. That made it possible to demonstrate in the form of geometrical models the implementation of pulse jet engines as the elements of influence on the processes of unfolding a rod structure.

3. Using a computer animation, we have determined the mutual arrangement of links in the four-link rod structures obtained as a result of the inertial unfolding of respective links using the pulse jet engines. An analysis of video frames of the animation has made it possible to choose the required point in time to fix (stop) the unfolding when the mutual arrangement of links in rod structures, required for use, occurs, as well as to calculate the current values of angles between the links.

4. Based on the inertial unfolding of four-link rod structures, we demonstrate schemes of the action of manipulators to grip cylindrical bodies whose axes are parallel or perpendicular relative to the surface of a spacecraft. It is shown that the implementation of the inertial unfolding requires that the endpoints of links should be exposed to the pulses of pyrotechnic devices at magnitudes of **U′**={0.1, 1.9, 1.3, 2.5} conditional units. To ensure specific implementation, we calculated the time for unfolding the structure and determined the magnitudes of coordinates of the "stop-code" vector $\mathbf{U}_{\mathrm{STOP}}$.

5. The result of our study is the constructed plots of change over time in the function of the generalized coordinates, as well as the first and second derivatives from these functions; based on this, we estimated strength

characteristics of the system at the time of fixation (stopping) the unfolding. The derived phase trajectories of the unfolding process make it possible to estimate the speeds of elements in the structures when the unfolding brakes.

6. We have given test examples of the unfolding in weightlessness of certain variants of four-link rod structures such as a "robot's arm". The examples prove the possibility of geometrical modeling of objects in weightlessness based on the unfolding of the four-link rod structures when the drivers of the process are the pulse jet engines mounted at the endpoints of links in a rod structure.

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