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**MATHEMATICAL MODEL  
OF THERMAL FIRE DETECTOR WITH THE THERMISTOR**  
(presented by DSc Abramov Y.)

The article presents the results of research of thermistor design, composition and nominal resistance influence on dynamic parameters of thermal fire detector.

**Keywords:** fire detector, sensitive element, thermistor, dynamic equation, dynamic parameter, time constant.

**Problem formulation.** Fire detectors (FD) constitute the important part of modern fire alarm systems (FAS), documentation for which does not always contain their main technical data. Research of FD operation efficiency demands information on technical data of their components, in particular dynamic parameters of sensitive element (SE).

Dynamic parameters (FD) can be determined by using of detector mathematical model which considers the material of semiconductor, constructive design and range of operating temperatures. Accordingly there is a problem of technical data improvement of FAS elements dynamic parameters.

**Analysis of recent researches and publications.** In [1, 2] the models of SE point thermal FD are suggested considering constructive characteristics and conditions of fire expansion. However, proposed models demand precise mathematical description of SE and additional experimental data of the researched FD. In [3] there is the model of SE with posistor without considering the design and in [4] the model of SE with thermistor which takes into consideration the constructive peculiarities of SE. According to [4], dependence of thermistor resistance  $R_T$  on temperature, considering the material of semiconductor and design peculiarities of thermistor is

$$R_T = AT^b e^{B/T}, \quad (1)$$

where  $A$ ,  $b$ ,  $B$  – are constant that depend on composition of thermistor semiconductor material and sensitive element design;  $T$  – current thermistor temperature,  $K$ .

In [5] for simplifying of engineering calculations under  $b \ll 1$  it was accepted that constant  $A$  corresponds to thermistor resistance  $R_\infty$  under infinitely high temperature. At this in order to increase the precision and for convenience of  $R_T$  determination it is suggested to take into consideration the nominal resistance

$$R_T = R_H e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)}, \quad (2)$$

where  $R_H$  – thermistor nominal resistance, Ohm, under the temperature  $T_H$ , K.

Approach for determination of FD dynamic parameters according to test data is presented in [6].

**Statement of the problem and its solution.** This work is aimed on research of total influence of thermistor semiconductor material, SE design peculiarities and its nominal resistance under set temperature on the FD dynamic parameters. Let's perform mathematical description of FD with thermistor in the interval of working temperatures taking into account the SE design peculiarities, semiconductor material and its nominal resistance under set temperature. Equation (1) considering (2) has the following view

$$R_T = R_H T^b e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)}. \quad (3)$$

Let's compare differentials of the left and right part:

$$dR_T = \left[ R_H T^b e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)} \right]' dT; \quad (4)$$

$$\frac{dR_T}{dT} = bT^{(b-1)} R_H e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)} - T^b \frac{B}{T^2} R_H e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)}; \quad (5)$$

$$\frac{dR_T}{dT} = bT^{(b-1)} R_t + T^b \beta R_t = T^{(b-1)} R_t (b + T\beta); \quad (6)$$

$$R_t = R_H e^{\left(\frac{B}{T} - \frac{B}{T_H}\right)}; \quad \beta = -\frac{B}{T^2}, \quad (7)$$

where  $R_t$  – thermistor resistance under nominal temperature, Ohm;  $\beta$  – temperature coefficient.

Mathematical model of thermistor, as dynamic chain, is obtained from equation for non-stationary thermal exchange under criteria  $Bio < 0,1$  that corresponds to even distribution of temperature inside SE.

Quantity of heat transferred and absorbed by thermistor [4]

$$C \cdot m \cdot d \frac{dT}{d\tau} + \alpha F dT = \alpha F dT_B, \quad (8)$$

where  $C$  – thermistor material heat capacity,  $J \cdot kg^{-1} \cdot K^{-1}$ ;  $m$  – thermistor mass, kg;  $T$  – thermistor temperature, K;  $\tau$  – hour, sec;  $\alpha$  – coefficient of convection thermal exchange,  $W \cdot m^{-2} \cdot K^{-1}$ ;  $F$  – thermistor surface square,  $m^2$ ;  $T_B$  – environment temperature, K.

Let's insert (6) into (8) and get

$$\frac{C \cdot m}{R_t T^{b-1} (b + \beta T)} \cdot d \frac{dR_T}{d\tau} + \frac{\alpha \cdot F}{R_t T^{b-1} (b + \beta T)} \cdot dR_T = \alpha \cdot F \cdot dT_B. \quad (9)$$

In the result of research, after linearization (9) by the method of full differential the FD dynamic equation was obtained, which considers the overall impact of thermistor semiconductor material, SE design peculiarities and nominal resistance:

$$T_T \dot{\overline{r_T}} + \overline{r_T} = K_T \overline{t_B}; \quad (10)$$

$$T_T = \frac{C \cdot m}{\alpha \cdot F}; \quad K_T = \frac{T_{B0}}{R_{T0}} R_t T^{b-1} (b + \beta T), \quad (11)$$

where  $T_T$  – thermistor time constant, sec;  $K_T$  – thermistor amplification coefficient;  $\overline{r_T}$ ,  $\overline{t_B}$  – relative variables.

According to [6]. The FD time constant  $T_T$  is determined by formula

$$T_T = \frac{(t_1 - t_{stat})}{(dt/d\tau)_0 K_T}, \quad (12)$$

where  $t_1$  – FD trigger temperature for the programmed temperature growth speed, K;  $t_{stat}$  – static temperature of FD trigger, K;  $(dt/d\tau)_0$  – programmed temperature change speed [K/ sec].

Analysis of the obtained results demonstrates that in order to decrease the FD inertia, mass should be decreased and SE square increased. In order to improve the convective thermal exchange, plate radiator should be placed on SE.

**Conclusions.** FD dynamics equation was obtained considering the overall influence of thermistor semiconductor material, SE design peculiarities and its nominal resistance. In order to decrease FD inertia mass should be decreased and SE square increased.

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**Математическая модель теплового пожарного извещателя с термистором**

Представленные результаты исследования влияния конструктивных особенностей, состава и номинального сопротивления термистора на динамические параметры теплового пожарного извещателя.

**Ключевые слова:** пожарный извещатель, чувствительный элемент, термистор, уравнение динамики, динамический параметр, постоянная времени.

Р.Г. Мелешенко, В.О. Дуреев

**Математична модель теплового пожежного сповіщувача з термістором**

Представлені результати дослідження впливу конструктивних особливостей, складу і номінального опору термістора на динамічні параметри теплового пожежного сповіщувача.

**Ключові слова:** пожежний сповіщувач, чутливий елемент, термістор, рівняння динаміки, динамічний параметр, постійна часу.