Розроблено методи обчислення рекурентних діаграм в просторі зі скалярним добутком, які дозволяють вивчати властивості і особливості вектора станів різних за складністю динамічних систем природної та соціальної сфери. Новий науковий результат полягає в розробці науково-методичного апарату для обчислення рекурентних діаграм векторів станів систем у розширених на основі скалярного добутку метричних просторах. Запропоновані два методи обчислення рекурентних діаграм для векторів станів складних динамічних систем, які мають високу інформативність, помірну складність і універсальність щодо розмірності досліджуваного простору станів. На практиці запропоновані методи можуть використовуватися для обчислення і порівняння рекурентних діаграм станів досліджуваних систем в метричних просторах різної розмірності без додаткової нормування. Перевірка працездатності запропонованих методів проведена на основі експериментальних спостережень концентрацій формальдегіду, аміаку та оксиду вуглецю в атмосфері промислового міста. Встановлено, що при значеннях кутового розміру області 10° і 30° запропонований метод обчислення рекурентних діаграм має підвищену інформативність, меншу складність та інваріантність до розмірності простору станів. Показано, що методи обчислення рекурентних діаграм в просторі зі скалярним добутком дозволяють використовувати їх при наявності короткочасних інтервалів відсутності спостережень. Експериментально встановлено, що в окремих випадках параметрів результати обчислення рекурентних діаграм на основі розроблених методів збігаються з результатами відомих методів. Це свідчить про більш загальний характер запропонованих методів

Ключові слова: рекурентні діаграми, вектор станів, забруднення атмосфери, складні динамічні системи

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CONSTRUCTION OF METHODS FOR COMPUTING RECURRENCE PLOTS IN SPACE WITH A SCALAR PRODUCT

B. Pospelov
Doctor of Technical Sciences, Professor
Research Center*
E-mail: pospelov@nuczu.edu.ua

V. Andronov
Doctor of Technical Sciences, Professor
Research Center*

R. Meleshchenko
PhD
Department of Fire and Rescue Training*

Yu. Danchenko
PhD, Associate Professor
Department of General Chemistry
Kharkiv National University of Civil Engineering and Architecture
Sumska str., 40, Kharkiv, Ukraine, 61002

I. Artemenko
Doctor of Law, Associate Professor
Kharkiv National University of Internal Affairs
Lva Landau ave., 27, Kharkiv, Ukraine, 61080

M. Romaniak
PhD
Institute of Public Administration of Ukraine
Taras Shevchenko National University of Kyiv
Volodymyrska str., 60, Kyiv, Ukraine, 01033

A. Khmyrova
PhD
Educational and Scientific Production Center*

T. Butenko
PhD, Senior Research
Department of Organization and
Coordination of Research Activities
Scientific-Methodical Center of Educational Institutions
in the Sphere of Civil Defence
Chernyshevska str., 94, Kharkiv, Ukraine, 61023
*National University of Civil Defence of Ukraine
Chernyshevska str., 94, Kharkiv, Ukraine, 61023

1. Introduction

The study of complex systems of natural and artificial origin suggests that they are based on various nonlinear processes. The study of these processes is necessary for deep and comprehensive understanding and modeling of such systems. Traditional methods [1], based on the concept of linearity, have been recently considerably supplemented by different methods of the theory of nonlinear dynamics and chaos. However, most methods of nonlinear analysis are based either on rather long or stationary series of observation data that are difficult to get when observing actual natural phenomena and systems. In paper [2], it was shown that these methods give satisfactory results only for idealized models of actual systems. In this regard, there emerged the need to develop new methods for non-linear analysis of observation data for actual systems. One of such methods is based on the fundamental property of actual dissipative systems – recurrence (repeatability) of states. This property implies that in any complex dynamic system even a small perturbation can bring a system to an unstable state, accompanied by exponential deviation from its current state. However, after a while the system itself tends to come back to the state that is, in a sense, close to the previous one and in this case passes through similar stages of evolution. This behavior can be visualized in the form of relevant recurrence plots (RP), proposed in [3]. The popularity of the RP method with researchers is due primarily to its applicability to short and stationary data series. At present, the number of English-language studies on recurrent analysis exceeds several hundred. However, despite this, the problem of developing the methods of recurrent analysis is far from being solved and remains particularly relevant for researching various complex dynamic systems of natural and artificial origin. In this regard, one of the most important directions of development of the methods for recurrent analysis is further improvement of the methods of RP calculation for studying complex dynamic systems of various origins.

2. Literature review and problem statement

RP methods are based only on the results of actual observations of the states of complex dynamic systems with a priori unknown structure and properties. These methods are attractive for research into many complex dynamic systems of natural and artificial origin [4]. It is known that in general case the state of dynamic systems is characterized by some vector of finite size of their current states that change over time. In applications, the state vector is observed discretely over time. In this case, the observable vector of states is considered in the form of coordinates of a certain point moving along a trajectory in the corresponding phase space. Recurrence of the state of a dynamic system in this case means a situation, in which these points appear to be close in some ways at different moments of time [5]. In this case, it is convenient to display the recurrence of states in the form of RP [3]. The RP method, unlike the methods of correlation dimensionality [6], makes it possible to visualize the trajectories of states of complex dynamic systems in a 2-dimensional phase space. At that, the features of both the systems and turbulent influences are taken into account. In the studied systems, RP in a 2-dimensional phase space make it possible to identify visually their important properties, such as stochasticity, chaos, periodicity and quasi-periodicity of states, as well as the features of evolution and interrelation with other systems [3]. Under actual conditions, it is not always possible to observe all components of the vector of states of the studied dynamical systems. For complex systems, the size of the state vector is usually not known in advance and can change over time. However, interactions and their number in complex systems have such a character that it is possible to judge on the dynamics of the whole system even in the case when only one variable of the state vector is observed. This fact was stated as the Tackens theorem about the dimensionality of embedding [7]. However, an analysis of recurrence of states for only one observed variable of the vector of states without regard to the method of embedding carries more useful information about the properties of the studied complex dynamic system than other known methods. That is why this method is also widely used in various applications. Paper [8] proposes the method for predicting dangerous air pollution in industrial cities based on recurrence of states for only one variable of the vector of states of general pollution. In this case, the values of the observed variable of the states vector at fixed moments are considered as elements of a set (scalar magnitudes) of the metric space. A usual metric (distance) to R (space of all real numbers) is used as a metric. Other metrics and corresponding metric space to calculate the recurrence of states are not explored in the paper. Paper [3] deals with the vectors of states of the system and computation of states recurrence is carried out in normalized linear space, for which the norm of an arbitrary state vector is equal to the distance of the corresponding point from the origin of coordinates. The calculation of recurrence of the vector of states of data on wind velocity in five areas of Nigeria is limited to consideration of linear space with the Euclidean metric [9]. It is noted that the RP methods in the specified space allow identifying the areas and periods of the year, for which wind velocity proves to be favorable or unfavorable for using alternative sources of electricity generation. Paper [10] focuses on the problem of parameterization of RP computation under conditions of potential artifacts in the reconstructed state vector. In this case, the study is limited to exploration of the RP in the normalized space with the Euclidean metric. Other types of metrics are neither considered nor discussed. The procedure of calculation of RP for the restored vector of states by one-dimensional measurement of the complex dynamics in the Earth's magnetosphere in the scale of geo-space storms is discussed in article [11]. The RP is researched in the normalized linear space with maximum metric or the Chebyshev metric. Application of normalized space with the Euclidean metric to compute the matrices of recurrence and distances in neural networks is considered in research [12]. Features of calculating RP in normalized spaces with the Euclidean metric for analysis of the dynamics of states recurrence in biological systems are discussed in [13]. Paper [14] proposed to calculate RP in case of irregular data observations, an approach based on the use of metric spaces with metric in the form of the distance between the relevant observed data. It is pointed out that this approach appears useful for any method based on measuring distances, for example, the correlation method or the Lyapunov indicator assessment method. Paper [15] focuses on the exploration of multi-dimensional time series in biological systems based on the combination of multiple network approach with recurrent networks. However, the studies are based on the vector of embedding and are limited to its consideration in normalized space with the Euclidean metric. Other possible metrics that assign other space topologies are considered. Paper [16] addresses the development of new computational methods for analysis of data on drilling to study the specific features of a basic dynamic system of the ore body formation. In this case, the RP of drilling data are considered in normalized space of the general type with arbitrary metric without indicating the specific topology of the used space. Computation of RP for the vector of states of dynamic systems in the abstract phase space (trivial metric) is considered in [3]. The specific features of calculating RP for localization of transient processes are discussed in [17]. It is noted that the norm in phase space in principle can be determined by various metrics, for example, Euclidean [18] or angular metric [19], as well as the L1 norm. However, the influence of these methods for determining the norms on the result of RP calculation is not considered. Paper [20] addresses studying the RP of carbon monoxide concentrations at earlier ignitions in premises. Calculation of RP is limited to the set of the values of carbon monoxide concentrations considered in one-dimensional phase space with the usual metric of distances, as well as the relevant power matrix of distances. At this, calculation of RP in multidimensional phase spaces with other types of metrics is not considered. The methods and devices for data analysis in one-dimensional space without using the recurrence of states with a view to self-adjusting detection of fires are considered in [21]. It is noted that there is a problem determining the necessary threshold of starting [22].

Thus, the known methods for calculating RP are based on consideration of the evolution of the vector of states of the studied complex dynamical system in linear normalized spaces of finite dimensionality, the topological structure of which is determined by different types of metrics. The most explored are the methods in linear spaces with uniform, Euclidean and the maximum metric. At this, each of these metrics determines its own topology of the space. That is why, the calculation of RP in these spaces has different complexity, and the results turn out to be different. The methods for calculating RP in spaces with angular metric are much less explored. In addition, despite a widespread use of the RP methods to study complex dynamic systems of various origins, the known literature does not offer any methods for calculating RP in spaces with the improved topological structure. For example, an introduction of the additional geometric characteristic – scalar product of two vectors – can be considered as an improvement of the structure of the used metric spaces. In this regard, an important and unresolved part of the problem of improving the RP computation for studying complex dynamic systems of various origins is the development of the methods for their calculation in the space with scalar product.

3. The aim and objectives of the study

The aim of this study is to construct methods for computing recurrence plots in space with a scalar product to investigate the properties of various complex dynamic systems of natural and technical sphere.

To accomplish the aim, the following tasks have been set:

– to substantiate theoretically the methods for calculation of recurrence plots for the observed vector of states of a dynamic system in the space with scalar product;

– to verify experimentally the effectiveness of the proposed methods for calculation of recurrence plots using the example of observation of actual dynamics of the vector of states of hazardous pollutants of urban atmosphere.

4. Theoretical substantiation of the methods for calculation of recurrence plots in space with scalar product

The basis for calculating the RP of an arbitrary n-dimensional vector of the states of the studied complex dynamic system is usually a set R^n of ordered sequences n of real numbers. Different metric spaces can be formed based on such ordered sequences. If we assume that two arbitrary vectors of states of system $X^T=(x_1, x_2,..., x_n)$ and $Y^T=(y_1, y_2,..., y_n)$, the following functionals will determine the known metrics:

$$d_1(X,Y)=\sum_{i=1}^{n}|x_i-y_i|, \quad (1)$$

$$d_2(X,Y)=[\sum_{i=1}^{n}|x_i-y_i|^2]^{1/2}, \quad (2)$$

$$d_3(X,Y)=\max\{|x_i-y_i|;\ i=1,2,...,n\}. \quad (3)$$

Metrics (1) to (3) are usually referred to as uniform, Euclidean and maximum, respectively. The set of all X and Y with metric d form the corresponding metric space. It should be noted that two different metrics determined on one and the same set of elements will form different metric spaces and, respectively, different RP. If simple known algebraic relationships between the elements are introduced into the system of metric spaces, these spaces become linear spaces. In such spaces, it is possible to perform simple algebraic actions, such as algebraic actions with the studied vectors X and Y. Let us combine the geometric properties of metric and linear spaces by determining the real number characterizing the «size» of an element in linear space. In this case, this number will determine the norm of vector $\|X\|$, calculated using any mapping of linear space onto the actual axis. The norm usually meets the following requirements:

a) $\|X\|\geq 0$ and $\|X\|=0$, if only $X=0$;

b) $\|X+Y\|\leq\|X\|+\|Y\|$;

c) $\|\alpha X\|=|\alpha|\|X\|$.

Given these properties, it is possible to show that:

$$\|X-Y\|=d(X,Y). \quad (4)$$

This means that (4) is a metric, since all the conditions that apply to the notion of distance are met. Usually these are conditions of a nonnegative magnitude, regardless of the direction and obeying the rule for the lengths of the sides of a triangle. This metric is used in the normalized linear space, if it is necessary that the space should be metric. In this case, the norm of a vector is equal to the distance from a point to the origin of coordinates, and metrics (1) to (3) are obtained through the norms. For example, following metric (2), the norm of an arbitrary vector X will be determined by ratio:

$$\|X\|=\left[\sum_{i=1}^{n}|x_i|^2\right]^{1/2}. \quad (5)$$

In the general case, before calculating RP, it is necessary to pay attention to the choice of the norm in the corresponding space. This is due to the fact that the boundaries of the norm have different configurations that affect the form of the RP. Calculation of RP on observations of a certain m-dimensional vector of states Z_i of the studied dynamical system at any moment i is carried out in accordance with the expression:

$$R_{i,j}^{m,\varepsilon}=\Theta\left(\varepsilon-\|Z_i-Z_j\|\right),\ Z_i\in\Omega^m,\ Z_j\in\Omega^m,$$
$$i,j=0,1,2,...,N_S-1, \quad (6)$$

where $\Theta(*)$ is the Heaviside function; ε is the dimensionality of the neighborhood of a point for vector Z_i at moment i in the studied space; $\|*\|$ is the norm of the vector in given space; N_S is the maximum number of observations of the vector of states. This means that the RP method (6) shows the trajectories of the vector of states of a system in m-dimensional phase space on a two-dimensional binary matrix of size $N_s \times N_s$ consisting of unities and zeros. In this case, each element of the matrix that is equal to unity at moments i and j will determine the recurrence (repeatability) of the vector of the states of the studied system. Coordinate axes are corresponding discrete moments of observation. Following (6), the RP form for the specified observations will depend on the chosen norm of the linear normalized space and the size of neighborhood ε. At a fixed value of magnitude ε, the uniform norm (1) will determine the minimum number of neighboring points, the Euclidean norm (2) – the average number of points, and the maximum norm (3) – the highest number of points [23]. In this paper, it is noted that when calculating the RP, the maximum norm with metric (3) is usually used. This is explained by the fact that this norm does not depend on dimensionality of the vector of states (dimensionality of the phase space), is the simplest to calculate and makes it possible to study RP analytically.

Independence of this norm on dimensionality makes it possible to compare directly the RP, calculated for different phase spaces and dimensionalities of embedding, while for other norms, the comparison of RP requires appropriate scaling.

An important parameter when calculating RP is also the choice of the size of the neighborhood ε. If too small ε is chosen, there can be almost no recurrent conditions. In this case, it is impossible to learn anything about the structure of the studied system. On the other hand, if the chosen size ε is too large, almost every point of the state vector in phase space turns out to be recurrent to any other point. This will lead to the emergence of a large number of artifacts. That is why a compromise is required when choosing the size of the neighborhood ε. The existence of noise will distort the original structure. That is why the desire to reduce its impact on the RP may require selection of large size ε of neighborhood, at which the preservation of the original structure of RP for the studied system will be ensured under noisy conditions.

Currently, only some rules of choosing size ε of the neighborhood are known. For example, the size should make up a few per cent of the maximum diameter of a phase space [24]. At the same time, the size should not exceed 10 % of the average or maximum diameter of the phase space. It is proposed in [25] for non-stationary case to select size ε so that the density of recurrent points should be approximately 1 %. Another criterion for choosing ε takes into consideration that the measurement of the process is a composition of an actual signal and some observation noise with standard deviation σ [26]. To obtain the results that are similar to the absence of noise, we should choose magnitude ε of the neighborhood size that is five times as much as the standard deviation of the observed noise, that is $\varepsilon > 5\sigma$. It is noted that this criterion is valid for a wide class of observable processes.

Ambiguity of choosing neighborhood size ε when calculating RP, complexity and non-uniformity of the known metrics and related metric spaces reduce the constructive capacities of the methods for studying the states of real dynamical systems, described in literature. This gives rise to a search and development of constructive methods for studying complex dynamic systems based on RP calculations. The proposed approach implies calculating RP in the spaces of the improved structure based on the introduction of an additional geometric characteristic in the form of scalar product of two vectors. Scalar product is mapping of ordered pairs of vectors of linear space onto the actual axis. Let this mapping be designated as (Z_i, Z_j). We will assume that mapping (Z_i, Z_j) satisfies the following conditions:

$$(Z_i, Z_j) = (Z_j, Z_i), \quad (7)$$

$$(aZ_i + bZ_k, Z_j) = a(Z_i, Z_j) + b(Z_k, Z_j), \quad (8)$$

$$(Z_i, Z_i) \geq 0 \text{ and } (Z_i, Z_i) = 0, \text{ only if } Z_i = 0. \quad (9)$$

An important consequence of conditions (7) to (9) is that the magnitude of:

$$(Z_i, Z_i)^{0.5} = \|Z_i\|. \quad (10)$$

This means that scalar product is the norm in linear space, because it satisfies the above properties for the norm. In this case, the validity of the ratio, called Schwarz inequality is important for scalar product:

$$|(Z_i, Z_j)|^2 \leq (Z_i, Z_i)(Z_j, Z_j). \quad (11)$$

Following (10), a scalar product gives rise to a norm, which, in turn, in accordance with (4) gives rise to metric. Therefore, the improved space with scalar product becomes a metric space, if an appropriate particular metric (1) to (3) is introduced. It is proposed to use the resulting improved space for calculation of RP. At the same time, scalar product can be interpreted as a measure of an angle between the observed vectors of the states of the studied system. To calculate RP, it is proposed to use representation (11), which can be written in the equivalent form of:

$$|(Z_i, Z_j)| \leq \|Z_i\| \|Z_j\|. \quad (11)$$

Then to calculate the RP as a measure of angle $\Theta_{i,j}$ between the corresponding vectors of states Z_i and Z_j, we will choose the magnitude:

$$S_{i,j} = \cos(\theta_{i,j}) = \frac{(Z_i, Z_j)}{\|Z_i\| \|Z_j\|}. \quad (12)$$

Normally, the concept of vectors' orthogonality is used in a theoretical study. In the explored case of the RP computation, it is proposed to use magnitude (12) to determine the degree of recurrence of the states of dynamical systems. Considering (6), in the improved space with scalar product, the proposed method for RP calculation is reduced to determining the magnitude of:

$$RC_{i,j}^m = \Theta(\varepsilon - S_{i,j}),\ Z_i \in \Omega^m, Z_j \in \Omega^m,$$

$$i, j = 0, 1, 2, \ldots, N_S - 1, \quad (13)$$

where $\varepsilon = \cos(\pi\alpha/180)$, and magnitude α assigns the angular size of the neighborhood of recurrence of states in degrees for vector Z_i at moment i in the corresponding space. In this case, the size ε of neighborhood for RP calculation should be chosen only according to the condition of permissible angular proximity (magnitude α) of observed vectors of states. It is not required to take into account the used metric of space in this case. Magnitudes (12) and (13) do not depend on dimensionality and the length of vectors of states observed at different moments either. Complete coincidence of these vectors (magnitude $\alpha = 0°$) corresponds to magnitude $S_{i,j} = 1$.

That is why the size ε of neighborhood (magnitude α) when calculating the recurrence of the studied vectors of states will be determined by the specified region of proximity $S_{i,j}$ to unity (magnitude α=0°).

A possible modification of the method for RP calculation (13) can be the replacement of magnitude $S_{i,j}$ with some magnitude $C_{i,j}=(Z_i^{\mathrm{T}} Z_j)$. Magnitude $C_{i,j}$ is the scalar product of the corresponding vectors, which takes into consideration not only their angular differences but also the length of the vectors. In this case, $C_{i,j}$ is the length-angular identifier of the difference of corresponding vectors of states. Given this, the modified method for the RP calculation is reduced to determining the magnitude of:

$$RC1_{i,j}^{m}=\Theta(\varepsilon-C_{i,j}),\quad Z_i\in\Omega^m, Z_j\in\Omega^m,$$

$$i,j=0,1,2,...,N_S-1. \qquad (14)$$

In the modified method (14), size ε of the neighborhood for RP calculation can be an arbitrary fixed number. If in (14) we calculate the square root instead of $C_{i,j}$ from scalar product the difference between the corresponding vectors of states, the ratio (14) will enable calculating the RP, similar to method (6) when using the Euclidean metric of space. In this case, it is important that the Euclidean metric should not be used in the computation of RP.

5. Experimental verification of efficiency of the proposed methods for computing recurrence plots

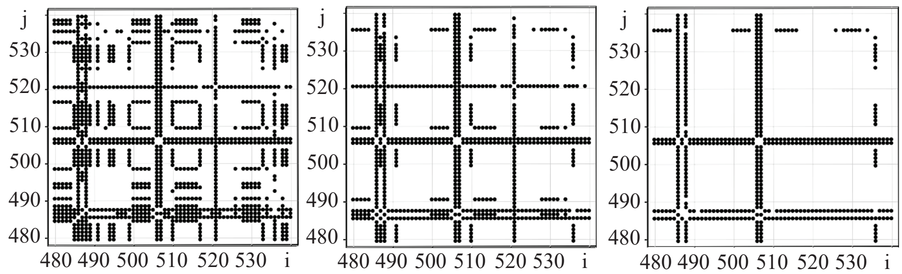
Verification of the efficiency of the proposed methods for RP calculation was based on the experimental data of the actual dynamics of the vector of states of hazardous gas pollutants of the urban atmosphere. The main sources of pollution of the atmosphere are vehicles [27], fires [28, 29] and accidents on the sites of critical infrastructure [30].

As it is known, global chemical pollution of the atmosphere causes the greenhouse effect, acid rains [31] and pollution of aquatic layers [32]. Formaldehyde, ammonia and carbon monoxide were chosen as specific studied components of the vector of states of atmospheric pollution. The procedure of the experiment was described in detail in [33].

The interval of the experiment lasting from 01:00 of May, 1, 2018 (i=480) till 01:00 of May, 15, 2018 (i=540) was chosen as the studied one. The RP (6) in spaces with different metrics (1) to (3) for the fixed magnitude ε, equal to 4 conditional units was assessed on this interval. The specified RP (6) for the vector of states of pollution, the components of which were normalized relative to the corresponding daily average maximum admissible concentrations are shown in Fig. 1 as an example.

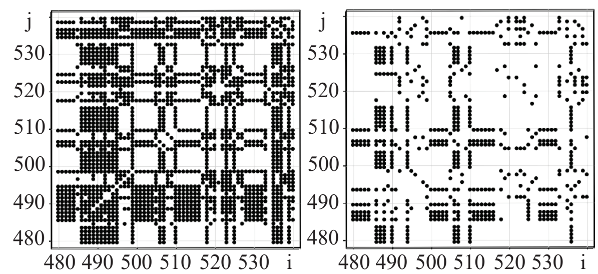
As an illustration, Fig. 2 shows the form of RP (13) for the studied vector of states of air pollution on the same test interval, but calculated in the proposed space with scalar product for cases if α=30° and α=10°.

The RP for the studied vector of states of the atmospheric pollution on the studied time interval, but calculated in accordance with the modified method (14), are shown in Fig. 3. The data in Fig. 3 correspond to fixed magnitude ε equal to 4 conditional units.



a *b* *c*

Fig. 1. RP of the studied vector of states of atmospheric air pollution on the test observation interval for metrics: *a* – d_1; *b* – d_2; *c* – d_3



a *b*

Fig. 2. RP of the studied vector of states of atmospheric air pollution on the test interval in the space with scalar product for different values of α: *a* – 30°; *b* – 10°

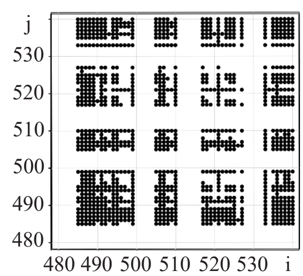


Fig. 3. The RP of the studied vector of states of atmospheric air pollution on the test interval in the space with scalar product, calculated with the use of the modified method (14)

Then, the proposed methods for RP calculation in the space with scalar product (13) and (14) were compared with the known method for RP calculation (6).

The results of comparison for the studied experimental data on pollution are presented in the form of the corresponding RP in Fig. 4.

To do this, arbitrary norm $\|Z_i-Z_j\|$, used in the known method (6), was represented following (10), instead of $S_{i,j}$ in method (13) or $C_{i,j}$ in method (14) through equivalent scalar product $\sqrt{(Z_i-Z_j)^{\mathrm{T}}(Z_i-Z_j)}$ of the difference of the corresponding vectors of the states of pollution. In this case, the RP in Fig. 4 corresponds to magnitude of neighborhood ε=4.

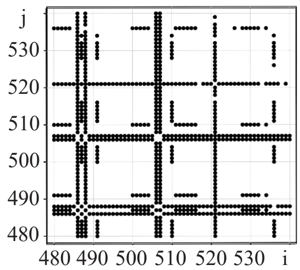


Fig. 4. RP of the studied vector of states of atmospheric pollution on the test interval in the space with scalar product in case when the norm in (6) is represented by magnitude $\sqrt{(Z_i - Z_j)^{\mathrm{T}}(Z_i - Z_j)}$

6. Discussion of results of experimental verification of the proposed methods for computing recurrence plots

Results of experimental verification of the proposed methods for the RP calculation in general indicate their functionality. Thus, for example, for the test interval of observation of the vector of states of pollution, calculated for known metrics (1) to (3), the RP and those presented in Fig. 1, are not the same. This fact does not contradict the known information that the metric of space affects the RP form. That is why the type of metric of space determines the corresponding form of the RP. According to Fig. 1, at the same size $\varepsilon=4$ of the neighborhood of the point for vector Z_i, a uniform metric (1) is the most informative. The maximum metric (3) should be considered less informative. Euclidean metric (2) occupies an intermediate position. At the same time, all the metrics (1) to (3) identify laminar states (vertical and horizontal sets of black dots) in the polluted atmosphere almost in the same way. On the interval between 500 and 510 counts, the vertical line of black dots corresponds to dangerous concentration of the studied contaminants that do not disperse in the atmosphere. The totality of white dots on this line characterizes a short-term loss of stability of the state of atmospheric processes with subsequent transition to the laminar state. The metrics have different computation complexity. In this case, only the maximum metric among the studied metrics is not dependent on dimensionality of space and makes it possible to compare the RP in spaces of different dimensionality without their prior normalizing.

Analysis of the RP of the vector of states of atmospheric pollution on the test interval in the space with scalar product for different values α of angular size of the region of recurrence of states, in general, indicates the functionality of the proposed method. In addition, this method is more informative than the method of maximum metric, less complex and does not depend on dimensionality of spaces of RP calculation. The method makes it possible to examine and compare the RP in spaces of different dimensionality without their prior normalizing.

The illustration of RP (Fig. 3), which were calculated based of the modified method (14) at magnitude ε equal to 4 conditional units, reveals the functionality and the possibility of the practical use of the method. At the same time, this method is easier to implement, because it is based only on calculating the scalar product of the corresponding vectors. In addition, the method for RP computation (14) does not depend on dimensionality of the studied vector of states and, in this sense, is universal. The implementation of the method does not require normalization of RP, which is necessary when using the known space metrics. At the same time, the proposed methods for RP calculation in space with scalar product possess a property that is important for applications. These methods can be used in cases of existence of intervals of absence of observation that are arbitrary in duration and time of occurrence. This property extends the scope of practical application of the proposed methods. In addition, it is not required to develop any additional special procedures for taking into account of intervals of data absence when calculating the RP.

The results of comparison of the proposed methods for calculation of RP in the space with scalar product and the known method (6) based of the calculation of arbitrary norm $\|Z_i - Z_j\|$ of the difference of corresponding vectors by calculation of scalar product $\sqrt{(Z_i - Z_j)^{\mathrm{T}}(Z_i - Z_j)}$, are shown in Fig. 4. The analysis of the data in Fig. 4 reveals that in the studied case, the RP turn out to be identical. However, the proposed method (14) proves to be easier than the known method (6) and results of the RP calculations are not dependent on the size of the vector of states. In this case, the RP in Fig. 4, calculated based of the method (14), is identical to the RP calculated based of method (6) when using the Euclidean metric.

In addition, the developed methods have certain limitations. The method (13) cannot be applied in the case of zero values of the lengths of the correspondent vectors of states Z_i and Z_j. Such situation can occur if there are specific gaps in measurement data, as well as when one of the state vectors has the zero length. Besides, this modified method (14) is free of such restrictions. It should be considered that an important limitation to methods (13) and (14) is the fact that neighborhood of recurrence for an arbitrary point, characterized by state vector Z_i at moment i, is represented in the form of some two-parameter region. One of the parameters of this region is determined by product of the lengths of the considered state vectors, and the other – by angular differences of these vectors.

7. Conclusions

1. The methods for calculating RP in space with scalar product, which allow exploring the properties and features of the vector of states of various by complexity dynamical systems of natural, technical, and social areas, were developed. The new scientific result is the development of the theoretical framework for the construction of new methods for the calculation of the RP of the vectors of states of the systems in metric spaces expanded on the basis of the scalar product. The proposed methods for RP calculation unlike the known methods are highly informative, have relatively low complexity, are universal in relation to the size of the state vector and can be applied when there are data absence intervals. This means that, in practice, such methods can be used to calculate and compare the RP of the studied systems in spaces of different dimensionality without additional RP normalization, which is necessary in the known methods, and when there are intervals of missing data.

2. The functionality of the proposed methods for calculating RP was verified on the basis of experimental observation of concentrations of formaldehyde, ammonia and carbon monoxide in the atmosphere of an industrial city with the typical configuration of buildings and existence of

pollution sources. The obtained results in general prove the functionality of the proposed methods. It was established that the proposed method for calculation of the RP of the vector of experimental states of atmospheric pollution on the test interval for angular dimensions α of the region of recurrence of states that are equal to 10° and 30° is more informative in comparison with the method of maximum metric. At the same time, the proposed method is less complex, as well as invariant to dimensionality of the space of states. It was proved that the proposed methods for calculating RP in the space with scalar product make it possible to use them when there are short-time intervals of the absence of observation. It was experimentally determined that in particular cases the results of the RP calculations based of the proposed methods coincide with results of the known methods. This indicates a more general nature of the developed methods for the RP calculation.

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